Term: Fall 2023

Instructor: Patricia Wrean

Name: Solution Set

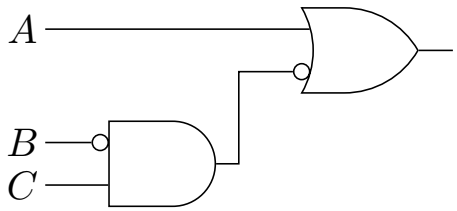
MATH 156
Test 2, Version A

Total = $\frac{\quad}{25}$

- **All of the work on this test must be your own.**
- You may use a scientific calculator. You may not use a calculator with graphing capability or a smartphone app.

GOOD LUCK!

1. (3 points) Write the Boolean expression that corresponds to the following gate diagram. Do not simplify!



$A + \overline{\overline{B}C}$

(-1) wrong and/or

(-1) wrong negations

(-½) each time you use non-Boolean symbol to max (-1)

2. (4 points) Use a truth table to simplify the logical expression $(p \oplus 1) \vee \sim(\sim q \vee 1)$.

p	q	$\sim q$	$p \oplus 1$	$\sim q \vee 1$	$\sim(\sim q \vee 1)$	$(p \oplus 1) \vee \sim(\sim q \vee 1)$
0	0	1	1	1	0	1
0	1	0	1	1	0	1
1	0	1	0	1	0	0
1	1	0	0	1	0	0

Conclusion: $\sim p$

(-½) per mistake for maximum (-1) per column

(-1) no conclusion

3. (3 points) The following statement is true: "If and only if I am tired, then I will order takeout." Given that, which of the following situations cannot occur?

   (a) I am not tired and I do not order takeout.

   (b) I am tired and I do not order takeout.

   (c) I am not tired and I order takeout.

p	q	p ⇔ q
0	0	1
0	1	0
1	0	0
1	1	1

⇔ must be both true or both false

so not possible means "exactly one of tired or takeout is true"

4. (3 points) Consider the following statements. Assuming that you know nothing about the number of students taking Math 156, answer the questions below. (Hint: zero is an even number. Also, there may be more than one correct answer.)

   (a) The number of students taking Math 156 is even or odd. $p \vee \sim p \Leftrightarrow 1$ complement

   (b) The number of students taking Math 156 is even and odd. $p \wedge \sim p \Leftrightarrow 0$

   (c) The number of students taking Math 156 is even or not odd. $p \vee p \Leftrightarrow p$ idempotent

   (d) The number of students taking Math 156 is even and not odd. $p \wedge p \Leftrightarrow p$

   Which of the above statements is always false? b

   Which of the above statements is always true? a

   Which of the above statements is the negation of "The number of students taking Math 156 is odd"?

   c, d

5. (2 points) The following statement is true: "If it is not winter, then magic beans will grow." Given that, answer the following by circling the correct choice.

   (a) Magic beans are growing. Is it winter? Yes / No / Maybe

   (b) It is summer. Will magic beans grow? Yes / No / Maybe

   (c) Magic beans are not growing. Is it winter? Yes / No / Maybe

   (d) It is winter. Will magic beans grow? Yes / No / Maybe

winter	beans	~winter	~winter → beans
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

6. (4 points) Is $p \leftrightarrow \sim q$ logically equivalent to $\sim(\sim p \rightarrow q)$? Use a truth table to justify your answer.

p	q	~p	~q	p ⇔ ~q	~p → q	~(~p → q)
0	0	1	1	0	0	1
0	1	1	0	1	1	0
1	0	0	1	1	1	0
1	1	0	0	0	1	0

Conclusion: No

For the questions on this page: if you are using the Laws of Logic, remember to use one law of logic per line, and be sure to state the name of the law you are using!

7. (4 points) Simplify the following using the laws of logic. If you're stuck, try using a truth table for part marks.

$$A(\overline{B}+B)+\overline{C}=\overline{A}\,\overline{C}+\overline{\overline{A}C}$$

$A(1) + \bar{C} = \bar{A}\bar{C} + \overline{\bar{A}C}$ complement

$A + \bar{C} =$ identity

$= \overline{\bar{A}\bar{C}} + A + \bar{C}$ De Morgan's

$= A + \bar{C} + \bar{C}$ absorption

$= A + \bar{C}$ idempotent

8. (2 points) Simplify the following. This is the nasty question I promised you and credit will only be awarded if the laws of logic are used to simplify the expression.

$$\sim(\sim(\sim p \vee \sim q) \vee \sim(\sim q \vee p))$$

method #1:

$(\sim p \vee \sim q) \wedge (\sim q \vee p)$ DeMorgans

$\sim q \vee (p \wedge \sim p)$ distributive

$\sim q \vee 0$ complement

$\sim q$ identity

method #2

$\sim((p \wedge q) \vee (q \wedge \sim p))$ DeMorgan's

$\sim(q \wedge (p \vee \sim p))$ dist.

$\sim(q \wedge 1)$ complement

$\sim q$ identity