

Term: Fall 2023

Name: Solution Set

Instructor: Patricia Wrean

MATH 156
Test 3, Version A

Total = $\overline{25}$

- All of the work on this test must be your own.
- You may use a scientific calculator. You may not use a calculator with graphing capability or a smartphone app. You may not share calculators between students.

GOOD LUCK!

1. (2 points) Label the following as “arithmetic”, “geometric” or “neither”.

(a) $1, 2, 6, 24, \dots$ neither

(b) $72, 36, 18, \dots$ geometric

2. (5 points) Consider the following.

$$a_n = 24 - 3n \quad \text{for } 3 \leq n \leq 17$$

- (a) Calculate the first three terms: 15, 12, 9

$$a_3 = 24 - 3(3) = 15$$

$$a_4 = 24 - 3(4) = 12$$

$$a_5 = 24 - 3(5) = 9$$

$\frac{1}{2}$ each

- (b) Calculate the final term: -27

$$a_{17} = 24 - 3(17) = -27$$

$\frac{1}{2}$

- (c) Give a recursive formula for a_n . Be sure to specify what values to use for the index.

Draw a box around your answer.

arithmetic with $d = -3$

$$\begin{cases} a_3 = 15 & \textcircled{1} \\ a_n = a_{n-1} - 3 & \textcircled{1} \text{ for } 4 \leq n \leq 17 & \textcircled{1} \\ & \text{or } 3 < n \leq 17 \end{cases}$$

$$\text{or } \begin{cases} a_0 = 15 \\ a_n = a_{n-1} - 3 \text{ for } 1 \leq n \leq 14 \end{cases}$$

$$\text{or } \begin{cases} a_1 = 15 \\ a_n = a_{n-1} - 3 \text{ for } 2 \leq n \leq 15 \end{cases}$$

3. (3 points) Consider the following:

$$\sum_{n=4}^{28} 3n = 12 + 15 + 18 + \dots + 84$$

(a) How many terms does it have?

$$k = n - m + 1 \\ = 28 - 4 + 1 = 25$$

25

(b) Evaluate the sum. Show your work below.

1200

arithmetic with $d=3$

$$S_k = \frac{k}{2} (a_m + a_n)$$

$$S_{25} = \frac{25}{2} (12 + 84) \\ = 1200$$

4. (3 points) Consider the following.

$$250 + 50 + 10 + \dots$$

$$\begin{array}{c} \curvearrowright \quad \curvearrowright \\ + \frac{1}{5} \quad + \frac{1}{5} \end{array}$$

it's a sum!

(a) Is this a sequence or a series? Choose one:

sequence / series

(b) Calculate the sum, if it exists. If it does not exist, say so and explain briefly. Show your work below.

312.5

geometric with $r = \frac{1}{5}$ ①

is $-1 < r < 1$? yes ✓

$$S_{\infty} = \frac{a_1}{1-r} = \frac{250}{1-\frac{1}{5}} = 312.5 \quad \text{①}$$

5. (3 points) Consider the arithmetic sequence with first term equal to 39 and final term equal to 225. The common difference is equal to 6. How many terms are in this sequence?

$$a_n = a_m + (n-m)d$$

$$225 = 39 + (n-1)6$$

$$186 = 6(n-1)$$

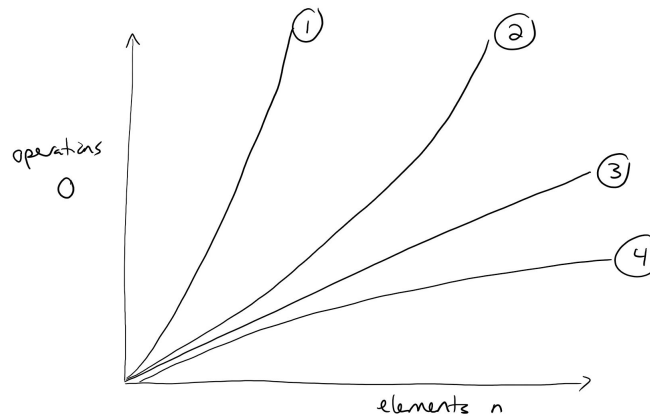
$$31 = n-1$$

$$n = 32$$

$$k = n - m + 1 = 32 - 1 + 1 = 32$$

There are 32 terms.

6. (2 points) Match the Big O notation with its corresponding curve on the graph. Please note that the curves are labeled 1, 2, 3, and 4 going from left to right and that curve 3 is a straight line.



- (a) $O(n)$
 (b) $O(n^2)$
 (c) $O(\log n)$
 (d) $O(n \log n)$

3
1
4
2

7. (2 points) Evaluate the following logarithms.

(a) $\log_{10}(0.1)$ $10^{-1} = 0.1$ -1

(b) $\log_4(64)$ $4^3 = 64$ 3

8. (2 points) For each of the following procedures, the number of operations needed for a task of size n is given below. Find Big O for each procedure.

(a) $3n^2 + 2n!$ $O(n!)$

(b) $(\log n)(4n + 5) = 4n \log n + 5 \log n$ $O(n \log n)$

$\left(\frac{-1}{2} \right)$ each time if only wrote 2^n , not $O(2^n)$, etc

9. (3 points) For a task of size n , Program A will always take one thousand steps to run and Program B will take $n \log n$ steps to run. Indicate whether the following statements are true or false.

- (a) Program B has logarithmic growth. True / False
- (b) There are no values of n for which Program B is a more efficient choice than Program A. True / False
- (c) If you think the task might have a very, very large n , Program A is probably a good choice. True / False