Term: Fall 2023

Instructor: Patricia Wrean

Name: Solution Set

MATH 156
Test 3, Version B

Total = $\frac{\quad}{25}$

- **All of the work on this test must be your own.**
- You may use a scientific calculator. You may not use a calculator with graphing capability or a smartphone app. You may not share calculators between students.

GOOD LUCK!

1. (2 points) Label the following as "arithmetic", "geometric" or "neither".

   (a) $25, 37, 49, \ldots$ (+12, +12) arithmetic

   (b) $1, 2, 6, 24, \ldots$ (×2, ×3, ×4) neither

2. (5 points) Consider the following.

$$a_n = 3^n \qquad \text{for } 2 \le n \le 12$$

   (a) Calculate the first three terms: 9, 27, 81

$a_2 = 3^2 = 9$

$a_3 = 3^3 = 27$

$a_4 = 3^4 = 81$

   (b) Calculate the final term: 531 441

$a_{12} = 3^{12}$

   (c) Give a recursive formula for a_n. Be sure to specify what values to use for the index. Draw a box around your answer.

geometric with $r = 3$

$$\begin{cases} a_2 = 9 & (1) \\ a_n = 3\,a_{n-1} & (1) \quad \text{for } 3 \le n \le 12 \text{ or } 2 < n \le 12 \end{cases} \quad (1)$$

or

$$\begin{cases} a_0 = 9 \\ a_n = 3\,a_{n-1} \quad \text{for } 1 \le n \le 10 \end{cases}$$

or

$$\begin{cases} a_1 = 9 \\ a_n = 3\,a_{n-1} \quad \text{for } 2 \le n \le 11 \end{cases}$$

3. (3 points) Consider the following:

$$\sum_{n=5}^{21} 4n = \underset{(5)}{20} + \underset{(6)}{24} + \underset{(7)}{28} + \ldots + \underset{(21)}{84}$$

(a) How many terms does it have? $k = n - m + 1 = 21 - 5 + 1 = 17$ 17 (1)

(b) Evaluate the sum. Show your work below. 884 (2)

arithmetic with $d = 4$

$$S_k = \frac{k}{2}(a_m + a_n)$$

$$S_{17} = \frac{17}{2}(20 + 84)$$

$$= 884$$

4. (3 points) Consider the following.

$$10 + 50 + 250 + \ldots \quad (\times 5, \times 5)$$

it's a sum

(a) Is this a sequence or a series? Choose one: sequence / series

(b) Calculate the sum, if it exists. If it does not exist, say so and explain briefly. Show your work below. DNE (1)

geometric with $r = 5$ (1)

$-1 < r < 1$? No!

does not exist

5. (3 points) Consider the arithmetic sequence with first term equal to 42 and final term equal to 162. The common difference is equal to 5. How many terms are in this sequence?

$a_n = a_m + (n-m)d$

$162 = 42 + (n-1)5$

$120 = 5(n-1)$

$24 = n-1$

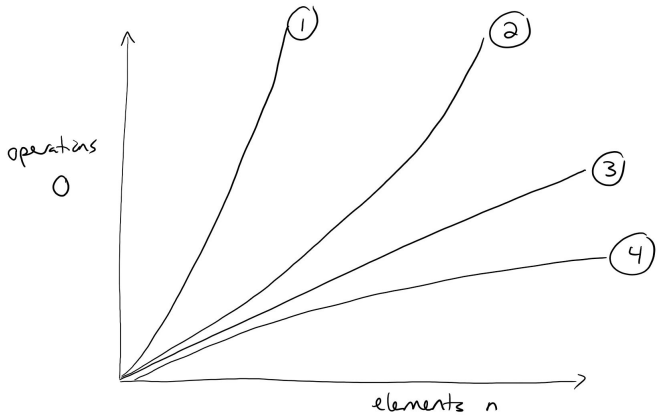
$n = 25$

Since I used $m=1$,

$k = n-m+1 = 25-1+1 = 25$

There are 25 terms.

6. (2 points) Match the Big O notation with its corresponding curve on the graph. Please note that the curves are labeled 1, 2, 3, and 4 going from left to right and that curve 3 is a straight line.



(a) $O(\log n)$ 4

(b) $O(n \log n)$ 2

(c) $O(n)$ 3

(d) $O(n^2)$ 1

7. (2 points) Evaluate the following logarithms.

   (a) $\log_2(64)$ $2^6 = 64$ 6

   (b) $\log_5(\frac{1}{5})$ $5^{-1} = 1/5$ -1

8. (2 points) For each of the following procedures, the number of operations needed for a task of size n is given below. Find Big O for each procedure.

   (a) $3n^2 + 2^n$ $O(2^n)$

   (b) $(\log n)(2n+1) = 2n\log n + \log n$ $O(n \log n)$

   (-½) each time if only wrote 2^n, not $O(2^n)$, etc

9. (3 points) For a task of size n, Program A will always take one thousand steps to run and Program B will take $n \log n$ steps to run. Indicate whether the following statements are true or false.

   (a) If you think the task might have a very, very large n, Program A is probably a good choice. True / False

   (b) There may be some values of n for which Program B is a more efficient choice than Program A. True / False

   (c) Program B has logarithmic growth. True / False

   ↑ it's "linearithmic"