Name: Solution Set

Signature: _____

Instructor: Patricia Wrean

Camosun College
Math Department

MATH 163
PRACTICE FINAL

Total = _____
100

You have three (3) hours to write this exam. *Please make sure that you have a complete exam before beginning.*

You may use your pen, pencil, eraser, ruler, protractor, and calculator. Scrap paper will be provided, but it **must be included** with your examination when you turn it in. You may **not** use your own scrap paper.

Your method of solution must be clearly shown and valid to obtain full marks for any problem.

GOOD LUCK!

1. Let A = {1, 3, 5, 7}, B = {2, 4, 6, 8}, U = {1, 2, 3, … 8}. State whether the following are T (true) or F (false). (4 points)

a) $A \cup B = U$ T

b) $5 \subseteq A$ $\in$ $5 \in A$, but 5 isn't a set, so cannot be a subset F

c) $A \cap \varnothing = B \cap \varnothing$ T

d) $\varnothing \subseteq U$ T

2. Given the sets A = {1, 2, 3}, B = {1, 3, 5, …}, and U = N, find the following.(3 points)

a) $\overline{A}$ $\{4, 5, 6, \ldots\}$

b) $\overline{B} \cap U$ $\overline{B} = \{2, 4, 6, \ldots\}$ $\overline{B} \cap U = \overline{B}$ $\{2, 4, 6, \ldots\}$

c) $\overline{A} \cap \overline{B}$ $\{4, 5, 6 \ldots\} \cap \{2, 4, 6 \ldots\}$ $\{4, 6, 8, \ldots\}$

3. Given that A = {1, 2, 3}, B = {2, 4, 7}, and U = {1, 2, 3, … 8}, use the **computer representation of sets** to find $\overline{\overline{A} \cup B}$. (3 points)

	1	2	3	4	5	6	7	8
A	1	1	1	0	0	0	0	0
B	0	1	0	1	0	0	1	0
$\overline{A}$	0	0	0	1	1	1	1	1
$\overline{A} \cup B$	0	1	0	1	1	1	1	1
$\overline{\overline{A} \cup B}$	1	0	1	0	0	0	0	0

$\overline{\overline{A} \cup B} = \{1, 3\}$

4. Let p denote "Harold lives in Victoria" and q denote "Harold lives in BC." Rewrite the following English sentences in terms of logical symbols (i.e. $p \wedge q$, $p \vee q$). Do not simplify! (4 points)

   a) Harold does not live in Victoria and he does not live in BC. $\bar{p} \wedge \bar{q}$

   b) It is not true that Harold does not live in Victoria. $\bar{\bar{p}}$

   c) Either Harold lives in Victoria or he does not live in BC, but not both. $p \oplus \bar{q}$

   d) If Harold lives in Victoria, then he lives in BC. $p \to q$

5. Complete a truth table for $A\bar{A}+B\cdot 1$, then give the simplest expression which is logically equivalent. (2 points)

A	B	$\bar{A}$	$A\bar{A}$	1	$B\cdot 1$	$A\bar{A}+B\cdot 1$
0	0	1	0	1	0	0
0	1	1	0	1	1	1
1	0	0	0	1	0	0
1	1	0	0	1	1	1

Same

$A\bar{A}+B\cdot 1 = B$

6. Is the negation of the conditional, $\overline{p \to q}$, logically equivalent to $p \wedge \bar{q}$? Use the following truth table to justify your answer. (Use as many or as few columns in the table as you wish.) (3 points)

p	q	$p \to q$	$\overline{p \to q}$	$\bar{q}$	$p \wedge \bar{q}$
0	0	1	0	1	0
0	1	1	0	0	0
1	0	0	1	1	1
1	1	1	0	0	0

yes $\overline{p \to q} \iff p \wedge \bar{q}$

7. Use the laws of logic to simplify the following logical expressions. If you're completely stuck, try using a truth table instead. (4 points)

a) $(p \wedge p) \vee (\bar{q} \vee 0) \vee (r \wedge \bar{r})$

$p \vee (\bar{q} \vee 0) \vee (r \wedge \bar{r})$ idempotent

$p \vee \bar{q} \vee (r \wedge \bar{r})$ identity

$p \vee \bar{q} \vee 0$ complement

$p \vee \bar{q}$ identity

$p \vee \bar{q}$

b) $\overline{A+B}(B+B\bar{C})$

$\overline{A+B}\ B$ absorption

$\bar{A}\,\bar{B}\ B$ de Morgans

$\bar{A}\ 0$ complement

0 identity

0

8. The following statement is true: If and only if the light is on, then it will use electrical power. Given that, answer the following with Y (yes), N (no), or M (maybe). (2 points)

a) The light is on. Does it use electrical power? Y
b) The light uses electrical power. Is it on? Y
c) The light does not use electrical power. Is it on? N
d) The light is off. Does it use electrical power? N

9. Find the following relation and state whether it is a function. (2 points)

Let $x \in \{1, 2, 3\}$ and $y \in \{1, 2, 3\}$. $(x,y) \in R$ if $x = y$.

$R = \{(1, 1), (2, 2), (3, 3)\}$

or

R

x	y
1	1
2	2
3	3

yes, it's a function because for every value of x, there is only one value for y.

10. Consider the various items you could find in a coffee shop. (6 points) 8

ITEM

Name	Type	Cost
brownie	dessert	3
coffee	drink	2
hot chocolate	drink	3
muffin	dessert	4
tea	drink	2
wrap	sandwich	5

PRICE

Cost	Range
2	low
3	medium
4	high
5	high

What would be the output of the following operations?

a) $\sigma_{\text{Type} = \text{"dessert"}}$ (ITEM)

Name	Type	Cost
brownie	dessert	3
muffin	dessert	4

b) $\pi_{\text{Name, Type}}(\sigma_{\text{Range} = \text{"medium"}}(\text{ITEM} \bowtie \text{PRICE}))$

Name	Type
brownie	dessert
hot chocolate	drink

Write an operation or sequence of operations to do the following:

c) Find the names (and only the names) of the items that cost $2.

$\pi_{\text{Name}}(\sigma_{\text{Cost} = 2}(\text{ITEM}))$

d) Find the names and costs of the items with a "high" range.

$\pi_{\text{Name, Cost}}(\sigma_{\text{Range} = \text{"high"}}(\text{ITEM} \bowtie \text{PRICE}))$

11. Let A={1,2} and B={2, 4, 6}. (3 points)

a) List the elements in the set B×A: $\{(2,1), (2,2), (4,1), (4,2), (6,1), (6,2)\}$

b) Is $A \times A \subseteq A \times B$? no

c) Is $B \subseteq B \times B$? no

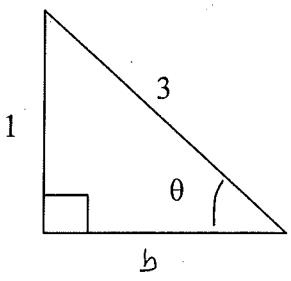
12. Use a calculator to evaluate the following. Round to two decimal places. (3 points)

$\tan 15.5°$ = 0.277325 → 0.28

$\sin(-565°)$ = 0.422618 → 0.42

$\cos^{-1}(2.5)$ → undefined

13. Find the **exact** values of the three basic trig functions of θ for the following triangle. Show your work. (4 points)



$a^2 + b^2 = c^2$

$1 + b^2 = 9$

$b^2 = 8$

$b = +\sqrt{8}$ ↑ only + sides allowed

$= 2\sqrt{2}$ ↑ simplified radical

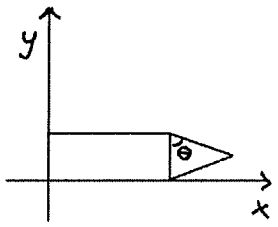
$\sin\theta = \frac{opp}{hyp} = \frac{1}{3}$

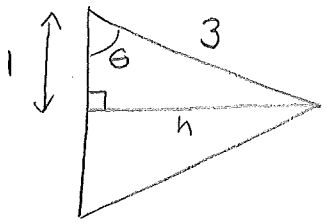
$\cos\theta = \frac{adj}{hyp} = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{opp}{adj} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

↑ must rationalize denominator

14. A Crayola crayon manufacturer wants to make a sign in the form of a crayon, as shown in the diagram below. The rectangle is 4 units long and 2 units wide. The vertical side of the triangle is also 2 units long, while the other two sides are both 3 units long. Calculate the coordinates of the pointed end of the crayon and the angle θ in the diagram. (4 points)





$a^2 + b^2 = c^2$

$h^2 + 1 = 9$

$h^2 = 8$

$h = \sqrt{8} = 2\sqrt{2}$

≈ 2.82843

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{3}$

$\theta = \cos^{-1}(1/3) \approx 70.5288°$

Coords of pointed end: (6.8, 1.0)

$\theta = 70.5°$ or $71°$

15. Give a formula for the *n*th term of each sequence. Also, state whether the sequence is arithmetic, geometric, or neither. (6 points)

125,114,103,92, . . . arithmetic with $d = -11$, $a_1 = 125$

$a_n = a_1 + (n-1)d$

$= 125 + (n-1)(-11)$

$= 125 - 11n + 11 = 136 - 11n$

arithmetic

$a_n = 136 - 11n$

or $\begin{cases} a_1 = 125 \\ a_n = a_{n-1} - 11 \end{cases}$

12, 6, 3, 3/2, . . . geometric $r = 1/2$, $a_1 = 12$

$a_n = a_1 r^{n-1}$

$= 12\left(\frac{1}{2}\right)^{n-1}$

geometric

$a_n = 12(1/2)^{n-1}$

or $\begin{cases} a_1 = 12 \\ a_n = \frac{1}{2} a_{n-1} \end{cases}$

$\left[= \frac{24}{2^n} \text{ if you insist} \right]$

16. Find the first three terms of the sequence $\begin{cases} a_1 = 3 \\ a_n = 4a_{n-1} + 5 \end{cases}$. (2 points)

$a_2 = 4 \cdot a_1 + 5$
$= 12 + 5$
$= 17$

$a_3 = 4 \cdot a_2 + 5$
$= 4 \cdot 17 + 5$
$= 73$

3, 17, 73

17. Find the sum of the following series. (3 points)

$\sum_{n=1}^{23} 8n+5 = (8 \cdot 1 + 5) + (8 \cdot 2 + 5) + \ldots$ (1) (2) (3) ... (23)

$= 13 + 21 + 29 + \ldots 189$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{23} = \frac{23}{2}(13 + 189)$

$= 2323$

2323

18. Evaluate, giving exact answers. (3 points)

$\log_{25} 5 = \log_{25} 25^{1/2}$

1/2

$\log_{25}(-5)$

undefined

$\ln e^{2x} = 2x (\ln e)^{=1}$

2x

19. Write as a single logarithm and simplify. Show your work. (5 points)

a) $\ln\frac{x^3}{y}+\ln\frac{y}{x^4}$ $= \ln\left(\frac{x^3}{y}\cdot\frac{y}{x^4}\right)$

$= \ln\left(\frac{1}{x}\right)$

$\ln(1/x)$

or $\ln x^{-1}$

or $-\ln x$

b) $\log_y(4y)-\frac{1}{2}\log_y 16$

1

$\log_y(4y) - \log_y 16^{1/2}$

$\log_y 4y - \log_y \sqrt{16}$

$\log_y 4y - \log_y 4$ → 1

$\log_y \frac{4y}{4}$

$\log_y y$

20. Solve the following equations, giving exact answers. (4 points)

$\log 0.01 = x$

$\{-2\}$

$0.01 = 10^x$

$10^{-2} = 10^x$

$x = -2$

$\log_x 27 = 3$

$\{3\}$

$27 = x^3$

$x = 3$

21. Solve the following equation, giving an exact answer and a decimal approximation to two decimal places. (3 points)

$2^x = 10$

method #1:

$\ln 2^x = \ln 10$

$x \ln 2 = \ln 10$

$x = \frac{\ln 10}{\ln 2}$

≈ 3.32193

method #2

$x = \log_2 10$

$= \frac{\log 10}{\log 2}$

$= \frac{1}{\log 2}$

≈ 3.32193

exact:

$x = \log_2 10 = \frac{\ln 10}{\ln 2} = \frac{1}{\log 2}$

approx:

$x \approx 3.32$

22. Phone numbers are of the form (###) ### – ####, where the three digits in the brackets are called the area code. (3 points)

a) How many possible area codes are there if there are no restrictions?

10 10 10

1000 area codes

b) How many possible area codes are there if the middle digit has to be either a zero or a one?

10 2 10

200 area codes

23. Consider the following data set and calculate the desired quantities. (3 points)

68, 58, 12, 72, 54, 72

ordered: 12, 54, 58, 68, 72, 72

average = 63

mean = 56

median = 63

range = 72 - 12 = 60

24. A data set has mean equal to 60 and standard deviation equal to 8. (4 points)

a) If the data set is NOT mound-shaped, what can we say about the proportion of measurements that lie between 44 and 76?

$60 - 44 = 16$ ← 2 standard deviations (as is $76 - 60$)

Tchebysheff says that ≥ 75% will lie within 2s.

b) What is the name of the rule or theorem that we are using in part (a)?

Tchebysheff

c) If the data set IS mound-shaped, what can we say about the proportion of measurements that lie between 44 and 76?

~95% will fall in that range

d) What is the name of the rule(s) or theorem(s) that we are using in part (c)?

Empirical Rule

25. Fruitzilla Tropical Drink Co. is a company with 40 employees. (4 points)

a) A committee of 5 employees is to be formed. How many possible committees are there?

$${}_{40}C_5 = 658,008$$

b) The top three employees are to be chosen, and ranked first, second and third. How many possible rankings are there?

$${}_{40}P_3 = 59280$$

26. On the first day of December, you deposit one cent into your piggy bank. Suppose that on each day of December after that, you deposit twice as much as on the previous day. How much will you have in the bank after the last deposit? (Do you think your piggy bank will hold that number of pennies?) (5 points)

pennies: $1 + 2 + 4 + \ldots a_n$

31 terms

geometric: $r = 2$, $a_1 = 1$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$= \frac{1(1 - 2^{31})}{1 - 2}$$

$$= 2,147,483,647$$

$$= 2.1 \text{ billion pennies}$$

(My piggy bank isn't that big!)

27. In 1969, Dr. Evil demands one million dollars from the UN, and in 1997, he asks the UN for one hundred billion dollars. Assuming continuous compounding, (6 points)

a) what is the rate of growth for evil villain extortion demands?

b) if the next Austin Powers movie comes out in ~~2009~~ 2014 as claimed, how much money should Dr. Evil be demanding from the UN then?

a) $A = Pe^{rt}$

$100\,000\,000\,000 = 1000\,000\,e^{r \cdot 28}$

$10^{11} = 10^6 e^{28r}$

$10^5 = e^{28r}$

$\ln 10^5 = 28r$

$r = \frac{\ln 10^5}{28}$

≈ 0.411176

b) $A = Pe^{rt}$

$= 1000000\, e^{0.411176(45)}$

$= 1.08571 \times 10^{14}$

$= 109$ trillion dollars

(1.1×10^{14} is a perfectly acceptable answer)

Evil villian extortion demands are growing at a rate of 41% per year. (That's evil!) Dr Evil should demand 109 TRILLION DOLLARS!