

Section 1.12: The Biconditional

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9:59 AM

We often say:

"If you get 90% or higher in Math 163, then you will get an A+."

when we actually mean

"If and only if you get a 90% or higher in Math 163, then you will get an A+."

another way of saying it is:

"If you get a 90% or higher in Math 163, then you will get an A+ and vice versa."

what does it mean?

"if p , then q AND if q , then p "

or "if p , then q AND if \bar{p} , then \bar{q} "

or "either p and q are both true OR they are both false" → XOR

notation:

$$p \leftrightarrow q$$

"if and only if p , then q "

"iff p , then q "

"if p , then q , and vice versa"

example: Consider the following conditional statements. Do they still make sense when written as a biconditional?

① If the car battery is drained, then the car will not start. No

(because if the car won't start, there are usually a number of reasons that could be true)

② If two lines are perpendicular, then they meet at a right angle. Yes

example: Is the biconditional $(p \leftrightarrow q)$ logically equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$? Use a truth table to justify your answer.

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	1	1	1

← same →

YES

$$(p \leftrightarrow q) \Leftrightarrow ((p \rightarrow q) \wedge (q \rightarrow p))$$