Section 3.2: cont'd

Monday, November 02, 2015
10:33 AM

arithmetic series:

$$2 + 5 + 8 + \dots$$

notation: S_n = sum of the first n terms
(n^{th} partial sum)

example: consider $2 + 5 + 8 + \dots$

calculate S_8

$$S_8 = \underbrace{2 + \overbrace{5 + 8 + 11 + 14 + 17 + 20}^{25} + 23}_{25}$$

$$S_8 = 4 \cdot 25 = \underset{\substack{\uparrow \\ \text{number} \\ \text{of} \\ \text{pairs}}}{4} \cdot \underbrace{(2 + 23)}_{\substack{\text{sum of each and} \\ \text{also sum of first \& last}}}$$

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$

for n = even
or odd

$$S_7 = \underbrace{2 + \overbrace{5 + 8 + 11 + 14 + 17}^{22} + 20}_{22}$$

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for arithmetic sequences and series:

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

on formula sheet

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example: find the sum of the first 50 terms of $2+5+8+\ldots$

$$S_n = \frac{n}{2}(a_1 + a_n) \qquad a_n = a_1 + (n-1)d$$

$$= \frac{50}{2}(2 + 149) \qquad a_{50} = 2 + 49 \cdot 3 = 149$$

$$= 3775$$

example: evaluate

$$\sum_{k=4}^{50}(6k-3) = \underset{k=4}{(6\cdot4-3)} + \underset{k=5}{(6\cdot5-3)} + \underset{k=6}{(6\cdot6-3)} + \ldots \underset{k=50}{(6\cdot50-3)}$$

$$= 21 + 27 + 33 + \ldots 297$$

arithmetic with $a_1 = 21$
$d = 6$
$n = \text{last} - \text{first} + 1 = 50 - 4 + 1 = 47$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$= \frac{47}{2}(2\cdot21 + 46\cdot6)$$

$$= 7473$$