Review: Radicals

Thursday, November 05, 2015
10:32 AM

simplifying radicals:

radical : square root, cube root, n^{th} root

rules: a radical is fully simplified if and only if:

1. there are no perfect powers under the radical sign
   - there are no perfect squares under the square root sign
2. there are no radicals in the denominator
   - no fractions under square root sign

examples: are the following radicals simplified?

1. $\sqrt{81}$ no because $81 = 9^2$
2. $\frac{1}{\sqrt{3}}$ no, bc radical in denom
3. $\sqrt{5}$ yes
4. $\sqrt{\frac{3}{2}}$ no, bc radical in denom
5. $\sqrt{12}$ $\sqrt{12} = \sqrt{3 \cdot 4}$ and 4 is a perfect square

two nice properties of radicals:

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ for a, b non-negative real numbers

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ for a, b non-negative real numbers <u>and</u> b non-zero

---

simplify:

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{100} = 10$

$\sqrt{81} = 9$

simplify:

$\sqrt{8} = \sqrt{2 \cdot 4} = \sqrt{2} \cdot \sqrt{4} = 2\sqrt{2}$

$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\,\sqrt{3} = 2\sqrt{3}$

$\sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$

$\sqrt{98} = \sqrt{2 \cdot 49} = 7\sqrt{2}$

$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$

$\sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$

$\sqrt{48} = \sqrt{4 \cdot 12} = \sqrt{4 \cdot 4 \cdot 3} = 4\sqrt{3}$

or $\sqrt{48} = \sqrt{2 \cdot 24} = \sqrt{2 \cdot 2 \cdot 12} = \sqrt{2 \cdot 2 \cdot 2 \cdot 6}$

or $\sqrt{48} = \sqrt{2\cdot 24} = \sqrt{2\cdot 2\cdot 12} = \sqrt{2\cdot 2\cdot 2\cdot 6}$

$= \sqrt{2\cdot 2\cdot 2\cdot 2\cdot 3}$

$\sqrt{13} = \sqrt{13}$ (already simplified)

$\sqrt{21} = \sqrt{21}$

---

rationalizing the denominator

$$\frac{1}{\sqrt{3}}\,\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sqrt{\frac{2}{5}}\,\sqrt{\frac{5}{5}} = \frac{\sqrt{10}}{5}$$

$$\frac{1}{\sqrt{12}} = \frac{1}{\sqrt{4\cdot 3}} = \frac{1}{2\sqrt{3}}\,\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2\cdot 3} = \frac{\sqrt{3}}{6}$$

<u>but</u> if you insist

$$\frac{1}{\sqrt{12}}\,\frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{12}}{12} = \frac{\cancel{2}\sqrt{3}}{\cancel{12}\,6} = \frac{\sqrt{3}}{6}$$

$$\frac{7}{\sqrt{18}} = \frac{7}{\sqrt{9\cdot 2}} = \frac{7}{3\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{7\sqrt{2}}{6}$$