Section 1.6: Logical Equivalence

Tuesday, September 30, 2014
3:12 PM

consider the propositions p & q:

⇒ there are four combinations of the possible values of p & q since each of them can be either true or false

truth table (extremely long-winded version)

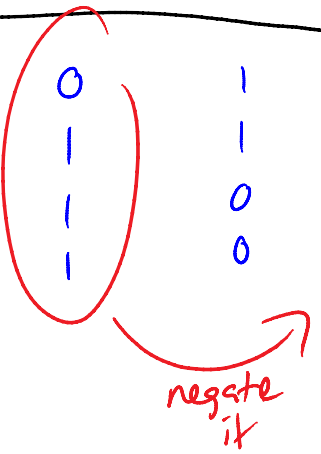
p	q	$p \wedge q$	$p \vee q$
false	false	false	false
false	true	false	true
true	false	false	true
true	true	true	true

(less longwinded version uses T and F instead of true and false)

really terse version: let 0 = false
1 = true

p	q	$p \wedge q$	$p \vee q$	$\bar{p}$	$\overline{p \vee q}$

0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	0
1	1	1	1	0	0



example: write the truth table for $p \oplus q$

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

example: write the truth table for $p \wedge \bar{p}$

p	$\bar{p}$	$p \wedge \bar{p}$
0	1	0
1	0	0

now, use that truth table to simplify

$p \wedge \bar{p}$:

$p \wedge \bar{p} \iff 0$

"is logically equivalent to"

example: use a truth table to simplify $p \wedge 1$

p	1	$p \wedge 1$
0	1	0
1	1	1

same

conclusion:

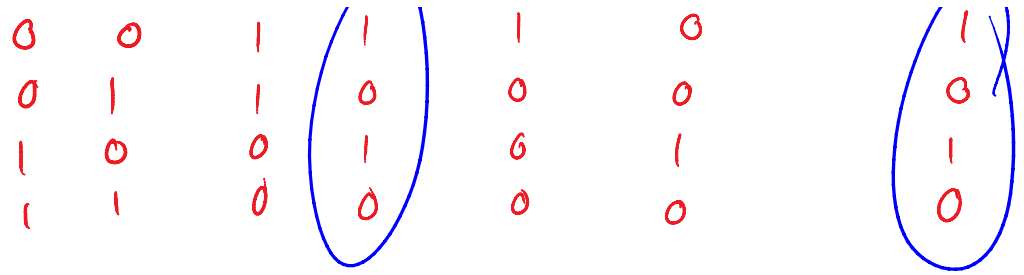
$$p \wedge 1 \Leftrightarrow p$$

example: write the truth table for $\overline{p \vee \bar{q}} \wedge \bar{r}$

p	q	r	$\bar{q}$	$\bar{r}$	$p \vee \bar{q}$	$\overline{p \vee \bar{q}}$	$\overline{p \vee \bar{q}} \wedge \bar{r}$
0	0	0	1	1	1	0	0
0	0	1	1	0	1	0	0
0	1	0	0	1	0	1	1
0	1	1	0	0	0	1	0
1	0	0	1	1	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	1	1	0	0
1	1	1	0	0	1	0	0

simplify $(\bar{p} \wedge \bar{q}) \vee (p \wedge \bar{q})$

p	q	$\bar{p}$	$\bar{q}$	$\bar{p} \wedge \bar{q}$	$p \wedge \bar{q}$	$(\bar{p} \wedge \bar{q}) \vee (p \wedge \bar{q})$
0	0	1	1	1	0	1
0	1	1	0	0	0	0



simplifies to $\bar{q}$

Is $\overline{p \oplus q}$ logically equivalent to $\bar{p} \oplus \bar{q}$?

p	q	$\bar{p}$	$\bar{q}$	$p \oplus q$	$\bar{p} \oplus \bar{q}$	$\overline{p \oplus q}$
0	0	1	1	0	0	1
0	1	1	0	1	1	0
1	0	0	1	1	1	0
1	1	0	0	0	0	1

NO

or $\overline{p \oplus q} \not\equiv \bar{p} \oplus \bar{q}$