Section 1.7: The Algebra of Sets

Wednesday, October 01, 2014
2:30 PM

membership tables

consider the element x in universe U
and the set A " " "

$\Rightarrow$ two possibilities: $x \in A$
$x \notin A$

example: draw the membership table for $A \cap \bar{A}$

$x \in A$	$x \in \bar{A}$	$x \in (A \cap \bar{A})$
no	yes	no
yes	no	no

extremely long-winded version

or

A	$\bar{A}$	$A \cap \bar{A}$
0	1	0
1	0	0

terse version

if we were to use this table to simplify $A \cap \bar{A}$, we would write:

$$A \cap \bar{A} = \emptyset$$

note: this must be a set

example: write out the membership table for $\bar{A} \cup B$

A	B	$\bar{A}$	$\bar{A} \cup B$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	0	1

example: Are the sets $\bar{B} \cup (B \cap \bar{A})$ and $\bar{A} \cup \bar{B}$ equal to each other?

A	B	$\bar{A}$	$\bar{B}$	$\bar{A} \cup \bar{B}$	$B \cap \bar{A}$	$\bar{B} \cup (B \cap \bar{A})$
0	0	1	1	1	0	1
0	1	1	0	1	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

Yes.

$$\boxed{\bar{A} \cup \bar{B} = \bar{B} \cup (B \cap \bar{A})}$$

computer representation of sets

consider the following sets

$$A = \{1, 3, 5\}$$

$B = \{1, 4, 7, 10\}$

and $U = \{1, 2, 3, \ldots 10\}$

use the computer representation of sets to find

$$\bar{A} \cap \bar{B}$$

elements

bit strings

sets	1	2	3	4	5	6	7	8	9	10
A	1	0	1	0	1	0	0	0	0	0
B	1	0	0	1	0	0	1	0	0	1
$\bar{A}$	0	1	0	1	0	1	1	1	1	1
$\bar{B}$	0	1	1	0	1	1	0	1	1	0
$\bar{A} \cap \bar{B}$	0	1	0	0	0	1	0	1	1	0

so $\bar{A} \cap \bar{B} = \{2, 6, 8, 9\}$

example: consider the sets $A = \{2, 4, 5, 6, 7\}$,

$B = \{1, 2, 3, 7, 8\}$ and $U = \{1, 2, 3, \ldots 8\}$

use the computer representation of sets to find

$$\overline{\bar{A} \cap B} \cup A$$

	1	2	3	4	5	6	7	8
A	0	1	0	1	1	1	1	0
B	1	1	1	0	0	0	1	1
$\bar{A}$	1	0	1	0	0	0	0	1
$\bar{A} \cap B$	1	0	1	0	0	0	0	1
$\overline{\bar{A} \cap B}$	0	1	0	1	1	1	1	0
$\overline{\bar{A} \cap B} \cup A$	0	1	0	1	1	1	1	0

$$\overline{\bar{A} \cap B} \cup A = \{2, 4, 5, 6, 7\}$$

or

$$\overline{\bar{A} \cap B} \cup A = A$$