Section 1.9: Laws of Logic

Monday, October 06, 2014
8:32 AM

Hardcopy & Online Assignment #2
due on Tuesday, Oct 14th

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there are connections between sets/logic/Boolean algebra:

	and	or	"not"		
logic	$p \wedge r$	$p \vee r$	$\bar{p}$	F or 0	T or 1
sets	$A \cap B$	$A \cup B$	$\bar{A}$	$\emptyset$	$\mathcal{U}$
Boolean	AB	$A + B$	$\bar{A}$	0	1

we can show that

$$p \wedge 1 \Leftrightarrow p$$

by using a truth table:

p	1	$p \wedge 1$
0	1	0
1	1	1

similarly: $A \cap \mathcal{U} = A$ by membership table/ Venn diagram

$A \cdot 1 = A$ by truth table

these statements are true for all possible values of the variable (or all possible sets in the universe), so we call them <u>laws</u>

→ these ones, in fact, are called the <u>identity</u> <u>laws</u>

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identity: there are four of them:

$p \wedge 1 \Leftrightarrow p$

$p \vee 1 \Leftrightarrow 1$

$p \wedge 0 \Leftrightarrow 0$

$p \vee 0 \Leftrightarrow p$

but note that these statements are true for all possible variables:

since $p \vee 0 \Leftrightarrow p$

then $\bar{q} \vee 0 \Leftrightarrow \bar{q}$

$r \vee 0 \Leftrightarrow r$

$$☺ \vee 0 \Leftrightarrow ☺$$

$$(p \wedge q) \vee 0 \Leftrightarrow p \wedge q$$

Why do we care?

example: simplify the following using the laws of logic (LOL)

note: use one law per line and be sure to state the name of the law you are using

$$(p \wedge 0) \vee (p \wedge 1)$$

$0 \vee p$ — identity

p — identity

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idempotent

$$p \wedge p \Leftrightarrow p$$

$$p \vee p \Leftrightarrow p$$

which also means that $☺ \wedge ☺ \Leftrightarrow ☺$

$$\bar{r} \vee \bar{r} \Leftrightarrow \bar{r}$$

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complement

$$\bar{\bar{p}} \Leftrightarrow p$$

$$p \wedge \bar{p} \Leftrightarrow 0$$

$$p \vee \bar{p} \Leftrightarrow 1$$

example: simplify the following using the complement law:

① $q \wedge \bar{q} \Leftrightarrow 0$

② $ABC + \overline{ABC} = 1$

③ ☺ $\cup$ $\overline{\text{☺}}$ $= U$

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note: for this class, you may omit writing either the commutative law or the associative law or both as a separate step

example: simplify $0 \vee p$

nitpicker-from-hell solution: $p \vee 0$ commutative
p identity

totally acceptable Math 163 solution: p identity