Section 1.11: cont'd

Thursday, October 09, 2014
8:30 AM

$p \to q$

if p is true, then q must also be true

if p is false, the law does not apply

if q is false, then p must also be false

$\therefore$ can't have p true and q false at the same time

example: True: If Pat sleeps in, she will be late for class.

let p = "Pat sleeps in"
q = "Pat is late for class"

conditional: $p \to q$

If $p \to q$ is true, is the converse $q \to p$ also true?

converse: If Pat is late for class, then she slept in.

no! not logically equivalent!

truth tables:

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

not same

so $p \to q$ is not logically equivalent to $q \to p$

$$p \to q \not\Leftrightarrow q \to p$$

there are actually four of these:

conditional	$p \to q$
converse	$q \to p$
inverse	$\bar{p} \to \bar{q}$
contrapositive	$\bar{q} \to \bar{p}$

example: For the conditional ($p \to q$) below, write the corresponding contrapositive ($\bar{q} \to \bar{p}$).

"If Pat sleeps in, then she is late for class."

→ "If Pat is not late for class, then Pat did not sleep in."

truth table:

p	q	$\bar{p}$	$\bar{q}$	$p \to q$	$\bar{q} \to \bar{p}$	$\bar{p} \to \bar{q}$
0	0	1	1	1	1	1
0	1	1	0	1	1	0
1	0	0	1	0	0	1
1	1	0	0	1	1	1

inverse

same

So $p \to q \iff \bar{q} \to \bar{p}$

example: write the contrapositive $(\bar{q} \to \bar{p})$ for the following conditional $(p \to q)$:

If I live in Saanich or Esquimalt, then I live in BC.

$\Rightarrow$ If I do not live in BC, then I don't live in Saanich $\boxed{\text{AND}}$ I don't live in Esquimalt.

conditional: $(p \vee q) \to r$

contrapositive $\bar{r} \to \overline{p \vee q}$ DeMorgan's

$\overline{p \vee q} = \bar{p} \wedge \bar{q}$

the "or" form of the conditional:

p	q	$p \to q$	$\bar{p}$	$\bar{p} \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

same!

$$p \to q \iff \bar{p} \vee q$$

example: write the "or" form $(\bar{p} \vee q)$ of the conditional $(p \to q)$:

If Pat sleeps in, then she will be late for class

$\Rightarrow$ Either Pat did not sleep in or she will be late for class or both.

where do you see this in code?

pseudocode:

if $x > 3$, then $y = 4$
print y

question: if the output is "4", was $x > 3$?

Consider

$x = 5$
$y = 7$
if $x > 3$ then $y = 4$
print y

output: 4

$x = 2$
$y = 4$
if $x > 3$ then $y = 4$
print y

output: 4