Section 1.12: The Bicconditional
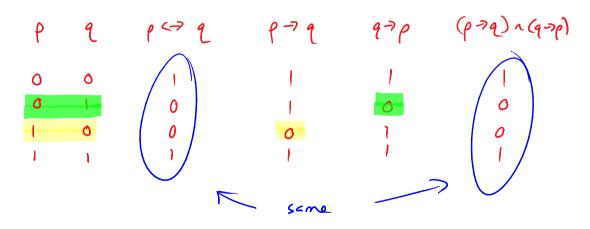
bicondition:

If you get 90% or higher in Math 163, then you will get an A⁺ and vice versa.

or

If and only if you get 90% or higher in Math 163, then you will get an A⁺.

what does it mean?

"if p, then q, and if q, then p"

or "if p, then q, and if $\bar{p}$, then $\bar{q}$"

or "either p and q are both true or they are both false"

notation:

$$p \longleftrightarrow q$$

"if and only if p, then q"

"if p, then q, and vice versa"

example: Consider the following statements. Do they still make sense when rewritten as a bicondition?

① If the car battery is drained, then the car will not start.

No

No

(because if the car will not start, there
are usually a number of reasons that
could be true)

② If two lines are perpendicular, then they
meet at a right angle.

Yes

③ If today is February 14th, then today
is Valentine's Day.

Yes

④ If you eat at Joe's, you will have a
good meal.

NO.

example: Is the biconditional $(p \Leftrightarrow q)$ logically
equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$?
Use a truth table to justify your answer

p	q	$p \Leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	1	1	1

same

Yes.

$p \Leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

example: If and only if Snarks are Boojums, then the Bellman is incorrect.

a) Snarks are Boojums. Is the Bellman correct? No
b) Snarks are not Boojums. Is the Bellman correct? Yes
c) The Bellman is correct. Are snarks Boojums? No
d) The Bellman is incorrect. Are Snarks Boojums? Yes