Section 2.1: Relations

ordered pair: (x, y)

examples: $(2,3), (4,-1)$

("Pat", "Math 163")

ordered triple: (x, y, z)

ordered n-tuple: (also known as "tuples")

("Ford", "Probe", 1990, "greg", "flaky electrical system")

("Math", "163", "X01", "Q1", 2014)

relations

relation ≡ a set of ordered pairs or n-tuples

how do you define a relation?

① list the n-tuples
② give a rule to generate the n-tuples

example of a relation:

$$x \mid y$$

x	y
0	4
1	8
2	-3
3	18

$\Leftrightarrow \quad \{(0,4), (1,8), (2,-3), (3,18)\}$

example:

let $x \in \{1, 2, 3\}$ and $y \in \{2, 4, 6\}$.

$(x, y) \in A$ if $x + y = 5$. Find A.

short method:

x	y
1	4
3	2

x	y
3	2
1	4

↖ either ↗

long method:

x	y
~~1~~	~~2~~
1	4
~~1~~	~~6~~
~~2~~	~~2~~
~~2~~	~~4~~
~~2~~	~~6~~
3	2
~~3~~	~~4~~
~~3~~	~~6~~

← write out all possible pairs and cross off the ones that don't fit the rule

example: Let $x \in \{1, 2, 3\}$ and $y \in \{2, 4, 6\}$.

$(x, y) \in A$ if $x \leq y$. Find A.

x	y
1	2
1	4
1	6
2	2
2	4
2	6
3	4
3	6

function $\equiv$ a relation which has for each value of the first component, there is only one value for the second component

"for every x, there is only one y"

examples: function?

x	y
-1	1
0	0
1	1

✓

x	y
1	-1
0	0
1	1

✗

instructor	course
Pat	Math 163
Pat	Math 172
Leah	Math 172

✗

Cartesian product:

example: let $A = \{0, 1, 2\}$ and $B = \{3, 4\}$

then $A \times B = \{(0,3), (0,4), (1,3), (1,4),$
$(2,3), (2,4)\}$

$B \times A = \{(3,0), (4,0), (3,1), (4,1),$
$(3,2), (4,2)\}$

$B \times B = \{(3,3), (3,4), (4,3), (4,4)\}$

definition: the Cartesian product $A \times B$ is the set of all ordered pairs where the first coordinate belongs to A and the second coordinate belongs to B

$\underset{=}{or}$ $A \times B = \{(x, y) \mid x \in A$ and $y \in B\}$

examples: let $X = \{0, 1, 2\}$, $Y = \{3\}$, and $Z = \{0, 3\}$

Find $Y \times Z$ $= \{(3,0), (3,3)\}$

$Y \times Z \times Y$ $= \{(3,0,3), (3,3,3)\}$

$Y \times Y \times Y \times Y = \{(3,3,3,3)\}$