Section 3.1: Sequences and Series

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9:47 AM

sequence ≡ an ordered list of numbers (often with a pattern)

examples:

		pattern?
①	2, 5, 8, ...	adding 3
②	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \frac{1}{256}$	multiplying by $\frac{1}{2}$
③	1, 4, 9, 16, 25, ... 100	n^2
④	5, -5, -15, ...	adding -10

notation:

$a_1, a_2, a_3, a_4, \ldots \quad a_n \ldots$

I will start with the subscript 1

what's the term before a_n?

a_{n-1}

three ways to define a sequence:

① list all of the terms or

enough of the terms to set up the pattern

note: if the sequence is finite, then you need to provide either the last term or the total number of terms

② give a general formula for a_n

③ give a recursive formula for a_n

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general formula: formula that gives the n^{th} term a_n in terms of n only

example: $\boxed{a_n = 3n - 1}$

what are the first three terms of the above sequence?

$a_1 = 3(1) - 1 = 2$
$a_2 = 3(2) - 1 = 5$
$a_3 = 3(3) - 1 = 8$

what is the 100^{th} term?

$a_{100} = 3(100) - 1 = 299$

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example: write all terms of the sequence

$\boxed{a_n = 2^{n+1} \quad \text{for} \quad 1 \leq n \leq 4}$

$a_1 = 2^{1+1} = 4$

$a_2 = 2^{2+1} = 8$

$a_3 = 2^{3+1} = 16$

$a_4 = 2^{4+1} = 32$

4, 8, 16, 32

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the recursive formula: gives the next term (or terms) by way of the previous ones

example: $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 3 \end{cases}$

$a_1 = 2$

$a_2 = a_1 + 3 = 2 + 3 = 5$

$a_3 = a_2 + 3 = 5 + 3 = 8$

the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, ...

$\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$ recursive formula

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digression: what is the general formula for this

sequence?

$$a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$