# Section 3.1: cont'd

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8:34 AM

series ≡ the sum of a sequence

example: $2 + 5 + 8 + \dots$

$5 + 15 + 25 + \dots \quad 105$

the sum of a finite sequence is just a number

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notation:

$S_n$ ≡ the sum of the first $n$ terms of a series
(if the series is finite, could be the sum of all of the terms)

→ aka "$n^{th}$ partial sum"

$S_\infty$ ≡ the sum of all terms in an infinite series

note: if $n$ is large, finding $S_n$ could be annoying! but we'll find more efficient methods later

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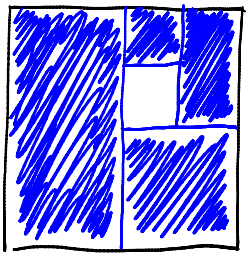
here's an interesting example:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 1$$



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consider the following series:

$$16 + 20 + 24 + \dots$$

for this series, calculate $S_3$ and $S_5$

$$S_3 = 16 + 20 + 24 = 60$$

$$S_5 = 16 + 20 + 24 + 28 + 32 = 120$$

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notation: Sigma notation

$$\sum_{n=1}^{4} (3n-1) = \underset{①}{(3-1)} + \underset{②}{(6-1)} + \underset{③}{(9-1)} + \underset{④}{(12-1)}$$

$$= 2 + 5 + 8 + 11$$

$$= 26$$

example: evaluate $\sum_{i=0}^{2} 3^i = \underset{①}{3^0} + \underset{②}{3^1} + \underset{③}{3^2}$

$$= 1 + 3 + 9$$

$= 13$

evaluate $\sum_{j=2}^{8} 7 = \underset{2}{7} + \underset{3}{7} + \underset{4}{7} + \underset{5}{7} + \underset{6}{7} + \underset{7}{7} + \underset{8}{7}$

$= 49$

how many terms? rule is:

# terms = last index − first index + 1

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write the following in sigma notation:

$\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \cdots \frac{1}{26} = \sum_{i=6}^{26} \frac{1}{i}$

$= \sum_{k=1}^{21} \frac{1}{k+5}$

$= \sum_{n=0}^{20} \frac{1}{n+6}$

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digression: why do we care?

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$7! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots$$