

Section 3.2: Arithmetic Sequences and Series

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8:34 AM

examples:

① $2, 5, 8, \dots$

② $0.4, 0.5, 0.6, \dots$

③ $5, -5, -15, \dots -205$

④ $-1, -\frac{3}{2}, -2, -\frac{5}{2}, \dots -10$

pattern?

add 3

add 0.1

add (-10)

add $(-\frac{1}{2})$

arithmetic sequence \equiv sequence in which you find the next term by adding a constant to the previous term

Common difference
 d

recursive formula

example: write the recursive formula for the sequence $2, 5, 8, \dots$

$$\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 3 \end{cases}$$

in general, arithmetic sequences can be written

$$\langle a_1 = \text{insert first term here} \rangle$$

$$\begin{cases} a_1 = \langle \text{insert first term here} \rangle \\ a_n = a_{n-1} + d \end{cases}$$

general formula: 2, 5, 8, ...

①	②	③	④	⑤	...	①
2,	5,	8,	11,	14,	...	a_n
2,	$2+3,$	$2+2 \cdot 3,$	$2+3 \cdot 3,$	$2+4 \cdot 3,$...	$2+(n-1) \cdot 3$

so, for this particular example

$$a_n = 2 + (n-1) \cdot 3$$

in general, $a_n = a_1 + (n-1) \cdot d$ for arithmetic

note: to find the simplified general formula for 2, 5, 8, ...

$$\begin{aligned} \text{we should simplify } a_n &= 2 + (n-1)(3) \\ &= 2 + 3n - 3 \\ &= 3n - 1 \end{aligned}$$

so $a_n = 3n - 1$ for 2, 5, 8, ...

what is the advantage of the general formula?

to find the 10,000th term, you plug 10,000
in for n

but with recursion, need to find the 9999th
term, and that requires the 9998th term, ...

example: find the general formula for the sequence
 $5, -5, -15, \dots$
be sure to simplify your answer

arithmetic with $a_1 = 5$
 $d = -10$

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ &= 5 + (n-1)(-10) \\ &= 5 - 10n + 10 \end{aligned}$$

$$a_n = 15 - 10n$$

example: for the arithmetic sequence in which
the first term is 2 and the 50th
term is 394, what is the common
difference?

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ 394 &= 2 + (50-1)d \\ 392 &= 49d \\ d &= 8 \end{aligned}$$

example: For the arithmetic sequence which has its
tenth term equaling 5 and its
twenty-fifth term equaling -70, give

the recursive formula for this sequence.

$$a_n = a_1 + (n-1)d$$

$$a_{10} = 5$$

$$5 = a_1 + 9d$$

$$a_{25} = -70$$

$$-70 = a_1 + 24d$$

system:
$$\begin{cases} 5 = a_1 + 9d \\ -70 = a_1 + 24d \end{cases}$$
 mult by -1

$$-5 = -a_1 - 9d$$

$$-70 = a_1 + 24d$$

$$-75 = 15d$$

$$d = -5$$

what's the first term?

$$5 = a_1 + 9(-5)$$

$$= a_1 - 45$$

$$a_1 = 50$$

recursive:
$$\begin{cases} a_1 = 50 \\ a_n = a_{n-1} - 5 \end{cases}$$