Section 3.3: cont'd

Wednesday, October 29, 2014
8:38 AM

recursive formula:

give a recursive formula for the sequence
100, 20, 4, 4/5, ...

$$\begin{cases} a_1 = 100 \\ a_n = \frac{1}{5} a_{n-1} \end{cases}$$

geometric
with $r = 1/5$

general formula: what's the general formula for
7, 14, 28, ... ?

1	2	3	4		n
7,	14,	28,	56,	...	a_n
7,	$7 \cdot 2$,	$7 \cdot 2^2$,	$7 \cdot 2^3$	...	$7 \cdot 2^{n-1}$

the general formula for 7, 14, 28, ...

is $\boxed{a_n = 7 \cdot 2^{n-1}}$

geometric sequences,

$$\boxed{a_n = a_1 \cdot r^{n-1}}$$

example: for the sequence 5, 15, 45, ..., find

a) the 12^{th} term

b) the 50^{th} term

a) $a_n = a_1 r^{n-1}$

$a_{12} = 5 \cdot 3^{11} = 885\,735$

b) $a_{50} = 5 \cdot 3^{49} = 1.196 \times 10^{24}$

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geometric series:

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}}$$

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why? digression

$S_n = a_1 + a_2 + a_3 + a_4 + \dots a_{n-1} + a_n$

$$\begin{cases} S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^{n-2} + a_1 r^{n-1} \\ -rS_n = -a_1 r - a_1 r^2 - a_1 r^3 - a_1 r^4 \quad -a_1 r^{n-1} - a_1 r^n \end{cases}$$

$S_n - rS_n = a_1 - a_1 r^n$

$(1-r)S_n = a_1(1-r^n)$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

example: find the sum of the first 12 terms of $2 + 10 + 50 + \ldots$

geometric $r = 5$

$a_1 = 2$

$n = 12$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{12} = \frac{2(1-5^{12})}{1-5}$$

$$= 122\ 070\ 312$$

(1.2×10^8 also acceptable)

by the way, $S_{50} = 4.4 \times 10^{34}$

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infinite geometric series:

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots$$

how to make this work? geometric with $a_1 = 1/4$, $r = 1/4$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_\infty = \frac{1/4\left(1-(1/4)^\infty\right)}{\ldots}$$

$(1 - 1/4)$

so, what's $\left(\frac{1}{4}\right)^{\infty}$?   let's consider $\left(\frac{1}{4}\right)^{n}$

as $n \to \infty$, $\left(\frac{1}{4}\right)^{n} \to 0$

but only for $-1 < r < 1$

or $|r| < 1$

then as $n \to \infty$, $r^n \to 0$

so $S_n = \frac{a_1(1 - \cancel{r^n}^{\,0})}{1-r}$ if $-1 < r < 1$

$$\boxed{S_\infty = \frac{a_1}{1-r}}$$

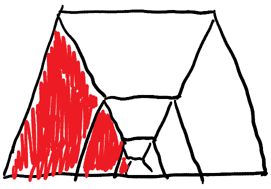
example: evaluate $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

geometric with $a_1 = \frac{1}{4}$ and $r = \frac{1}{4}$

$|r| < 1$?   ✓

$S_\infty = \frac{a_1}{1-r}$

$= \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}$



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evaluate:

a) $24 - 16 + \frac{32}{3} - \ldots$

geometric with $r = \frac{-2}{3}$ $\qquad |r| < 1$? ✓

$$S_\infty = \frac{a_1}{1-r}$$

$$= \frac{24}{1 + 2/3} = \frac{24}{5/3} = 24 \cdot \frac{3}{5} = \boxed{\frac{72}{5} = 14.4}$$

b) $12 + 18 + 27 + \ldots$

$$\cancel{S_\infty = \frac{a_1}{1-r} = \frac{12}{1-3/2} = \frac{12}{-1/2} = 12(-2) = -24}$$

$|r| < 1$? No

S_∞ = undefined

= does not exist (DNE)

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0 1 2

evaluate $\displaystyle\sum_{j=0}^{\infty} 75\left(\frac{3}{5}\right)^j = 75\left(\frac{3}{5}\right)^0 + 75\left(\frac{3}{5}\right)^1 + 75\left(\frac{3}{5}\right)^2 + \ldots$

$$= 75 + 75\left(\frac{3}{5}\right) + 75\left(\frac{3}{5}\right)^2$$

geometric with $a_1 = 75$

$r = \frac{3}{5}$ $\qquad |r| < 1$ ✓

$$S_\infty = \frac{a_1}{1-r}$$

$$= \frac{75}{1 - 3/5} = \frac{75}{2/5} = \boxed{187.5}$$

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repeating decimals:

$$0.\overline{4} = 0.4444444\ldots$$

$$= 0.4 + 0.04 + 0.004 + 0.0004 + \ldots$$

geometric: $a_1 = \frac{4}{10}$

$r = \frac{1}{10}$ $\qquad |r| < 1$? ✓

$$S_\infty = \frac{a_1}{1-r} = \frac{\frac{4}{10}}{1 - 1/10} = \frac{4/10}{9/10} = \frac{4}{10} \cdot \frac{10}{9} = \frac{4}{9}$$

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express $0.\overline{12}$ as a fraction:

$$0.\overline{12} = 0.12 + 0.0012 + 0.000012 + \ldots$$

geometric with $a_1 = 12/100$

$r = 1/100$ $\quad |r| < 1$ ✓

$$S_\infty = \frac{a_1}{1-r} = \frac{\frac{12}{100}}{1 - 1/100} = \frac{12/100}{99/100} = \frac{12}{100} \cdot \frac{100}{99} = \frac{12}{99} = \frac{4}{33}$$

so, what's $0.\overline{9}$?

geometric with $a_1 = 9/10$

$r = 1/10$ $\quad |r| < 1$ ✓

$$S_\infty = \frac{a_1}{1-r} = \frac{9/10}{1 - 1/10} = \frac{9/10}{9/10} = 1$$