Section 5.1: Cont'd

Tuesday, November 18, 2014
8:36 AM

Assignments #6 (online & hardcopy)
due Thursday Nov 27th

Test #3 on Tuesday, Dec 2nd
→ formula sheet (posted on web)

note: Thursday, Dec 4th is Test Makeup Day

Solving exponential equations: (start with nice straightforward type)

solve $2^{x-5} = 64$

$2^{x-5} = 2^6$

$x-5 = 6$

$\boxed{x = 11}$ or $\boxed{\{11\}}$

$2^{4-x} = \sqrt{\;}$

$$3^{4-x} = 3^{1/2}$$

$$4 - x = 1/2$$

$$4 - 1/2 = x$$

$$x = 3\tfrac{1}{2} \text{ or } 7/2 \text{ or } 3.5$$

$$\{7/2\}$$

why are we allowed to do this?

for $a > 0$ and $a \neq 1$,

~~if~~ iff $a^m = a^n$, then $m = n$

→ one-to-one property of exponential functions

"if the bases match in a equation, you can match the exponents"

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solve

$$9^{2x-1} = 3$$

$$9^{2x-1} = 9^{1/2} \qquad\qquad (3^2)^{2x-1} = 3$$

$$2x-1 = \frac{1}{2}$$
$$2x = \frac{3}{2}$$
$$x = \frac{3}{4}$$

$$3^{4x-2} = 3^1$$
$$4x-2 = 1$$
$$4x = 3$$
$$x = \frac{3}{4}$$

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application: compound interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where n = number of compounding periods per year

A = amount (final amount)

P = principal (initial amount)

r = interest rate

t = time of loan/investment

example: If \$1000 is deposited into an account paying 10% per year compounded monthly, how much will be in the account after 10 years?

10 years?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.1}{12}\right)^{12 \cdot 10}$$

$$= \$2707.04$$

how do you calculate the continuous case?

$$\boxed{A = Pe^{rt}}$$

where A = final amount
P = principal
r = interest rate
t = time

and "e" is a constant

$$e \approx 2.718281828459045\ldots$$

non-repeating, non-terminating decimal

irrational, like π and $\sqrt{2}$

so, \$1000 at 10% per year for 10 years:

$$A = Pe^{rt}$$

$$= 1000\, e^{0.1(10)}$$

$$= 1000\, e^{1}$$

digression:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots = \sum_{j=0}^{\infty} \frac{1}{j!}$$