

## Section 5.3: Properties of Logs

Wednesday, November 19, 2014  
9:29 AM

product rule:

$$\begin{aligned}\log 4.2 &= 0.623249 \\ \log 42 &= 1.623249 \\ \log 420 &= 2.623249 \\ \log (1000 \times 4.2) &= \log 4200 = 3.623249 = 3 + 0.623249 \\ &= \log 1000 + \log 4.2\end{aligned}$$

rule:  $\log_a (MN) = \log_a M + \log_a N$

note:  $a^m a^n = a^{m+n}$

examples: express as a single log, if possible

$$\ln (ab) + \ln (bc) = \ln (ab \times bc) = \ln ab^2c$$

$$\log 7 + \log 5 = \log 35$$

$$\log x^3 + \ln x^2 = \log x^3 + \ln x^2 \quad (\text{not same base})$$

simplify:  $\log 25 + \log 4$

$$\log 100$$

the power rule:

$$\begin{aligned}\log_a x^3 &= \log_a (x \cdot x \cdot x) \\ &= \log_a x + \log_a x + \log_a x \\ &= 3 \log_a x\end{aligned}$$

rule:

$$\log_a b^N = N \log_a b$$

simplify:

$$3 \ln x^2 - 2 \ln x^3$$

method #1

$$2 \cdot 3 \ln x - 3 \cdot 2 \ln x$$

$$6 \ln x - 6 \ln x$$

0

method #2

$$\ln (x^2)^3 - \ln (x^3)^2$$

$$\ln x^6 - \ln x^6$$

0

bad math:

~~$$23 \ln x - 32 \ln x - 9 \ln x$$~~

use the power rule to write an equivalent expression for

$$\ln x^7 = 7 \ln x$$

$$x \log_a b = \log_a b^x$$

$$\log \sqrt{7} = \frac{1}{2} \log 7$$

$$5 \log_2 9 = \log_2 9^5$$

$$\ln \frac{1}{m} = \ln m^{-1} = -\ln m$$

$$5 \log x^2 = \log x^{10} \text{ or } 10 \log x$$

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quotient rule:

$$\log_a \left( \frac{M}{N} \right) = \log_a M - \log_a N$$

digression: why?

$$\begin{aligned} \log_a \left( \frac{M}{N} \right) &= \log_a (MN^{-1}) \\ &= \log_a M + \log_a N^{-1} \\ &= \log_a M - \log_a N \end{aligned}$$

QED

use the quotient rule to expand the following

$$\log \frac{2}{x} = \log 2 - \log x$$

$$\ln \frac{5}{3} = \ln 5 - \ln 3$$

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write in terms of  $\ln 2$ ,  $\ln 5$ , and/or  $\ln x$ :

(your answer should look like  $\underline{\quad} \ln 2 + \underline{\quad} \ln 5 + \underline{\quad} \ln x$ )

$$\ln \frac{x}{2} = \ln x - \ln 2$$

$$\begin{aligned} \ln 8x^3 &= \ln 8 + \ln x^3 = \ln 2^3 + 3 \ln x \\ &= 3 \ln 2 + 3 \ln x \\ &= 3(\ln 2 + \ln x) \end{aligned}$$

$$\ln 10 = \ln (2 \cdot 5) = \ln 2 + \ln 5$$

$$\ln 50 = \ln (5 \cdot 5 \cdot 2) = 2 \ln 5 + \ln 2$$

$$\begin{aligned} \ln \sqrt{5x^2} &= \frac{1}{2} \ln 5x^2 \\ &= \frac{1}{2} [\ln 5 + \ln x^2] \\ &= \frac{1}{2} [\ln 5 + 2 \ln x] \\ &= \frac{1}{2} \ln 5 + \ln x \end{aligned}$$

$$\text{note: } \ln (ab)^2 = 2 \ln (ab)$$

$$\ln (ab^2) \neq 2 \ln (ab)$$

the base-change formula

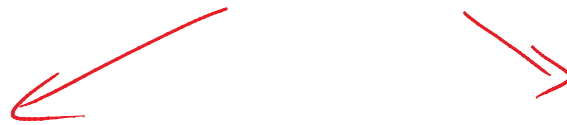
how can we calculate  $\log_3 7$  ?

↑  
note: no log-base-3 button  
on calculator

look at:

solve for  $x$ :

$$b^x = c$$



$$x = \log_b c$$

$$\log b^x = \log c$$

$$x \log b = \log c$$

$$x = \frac{\log c}{\log b}$$

← taking the  
log  
of both  
sides

$$\log_b c = \frac{\log c}{\log b}$$

base-change formula:

$$\log_b M = \frac{\log_a M}{\log_a b} \left( = \frac{\log M}{\log b} = \frac{\ln M}{\ln b} \right)$$

$$\log_3 7 = \frac{\log 7}{\log 3} \quad \text{or} \quad \frac{\ln 7}{\ln 3}$$

$$\approx 1.77124$$

So, calculate the following to 3 decimals:

$$\log_{22} 42 = \frac{\log 42}{\log 22} \approx 1.209$$

$$\log_{0.29} 103 = \frac{\ln 103}{\ln 0.29} \approx -3.744$$