Section 5.3: Properties of Logs

Wednesday, November 19, 2014
9:29 AM

product rule:

$\log 4.2 = 0.623249$

$\log 42 = 1.623249$

$\log 420 = 2.623249$

$\log(1000 \times 4.2) = \log 4200 = 3.623249 = 3 + 0.623249$

$= \log 1000 + \log 4.2$

rule: $\log_a(MN) = \log_a M + \log_a N$

note: $a^m a^n = a^{m+n}$

examples: express as a single log, if possible

$\ln(ab) + \ln(bc) = \ln(ab \times bc) = \ln ab^2c$

$\log 7 + \log 5 = \log 35$

$\log x^3 + \ln x^2 = \log x^3 + \ln x^2$ (not same base)

simplify: $\log 25 + \log 4$

$\log 100$

the power rule:

$$\log_a x^3 = \log_a (x \cdot x \cdot x)$$
$$= \log_a x + \log_a x + \log_a x$$
$$= 3 \log_a x$$

rule: $\boxed{\log_a b^N = N \log_a b}$

simplify: $3 \ln x^2 - 2 \ln x^3$

method #1

$$2 \cdot 3 \ln x - 3 \cdot 2 \ln x$$
$$6 \ln x - 6 \ln x$$
$$0$$

method #2

$$\ln (x^2)^3 - \ln (x^3)^2$$
$$\ln x^6 - \ln x^6$$
$$0$$

bad math: ~~$23 \ln x - 32 \ln x$~~
~~$-9 \ln x$~~

use the power rule to write an equivalent expression for

$\ln x^7 = 7 \ln x$

$x \log_a b = \log_a b^x$

$\log \sqrt{7} = \frac{1}{2} \log 7$

$5 \log_2 9 = \log_2 9^5$

$\ln \frac{1}{m} = \ln m^{-1} = -\ln m$

$5 \log x^2 = \log x^{10}$ or $10 \log x$

---

quotient rule:

$$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$$

digression: why?

$$\log_a \left(\frac{M}{N}\right) = \log_a (M N^{-1})$$

$$= \log_a M + \log_a N^{-1}$$

$$= \log_a M - \log_a N$$

QED

use the quotient rule to expand the following

$\log \frac{2}{x} = \log 2 - \log x$

$\ln \frac{5}{3} = \ln 5 - \ln 3$

$$\ln \frac{5}{3} = \ln 5 - \ln 3$$

write in terms of ln 2, ln 5, and/or ln x:

(your answer should look like __ln 2 + __ln 5 + __ln x)

$$\ln \frac{x}{2} = \ln x - \ln 2$$

$$\ln 8x^3 = \ln 8 + \ln x^3 = \ln 2^3 + 3 \ln x$$

$$= 3 \ln 2 + 3 \ln x$$

$$= 3(\ln 2 + \ln x)$$

$$\ln 10 = \ln (2 \cdot 5) = \ln 2 + \ln 5$$

$$\ln 50 = \ln (5 \cdot 5 \cdot 2) = 2 \ln 5 + \ln 2$$

$$\ln \sqrt{5x^2} = \tfrac{1}{2} \ln 5x^2$$

$$= \tfrac{1}{2} \left[\ln 5 + \ln x^2 \right]$$

$$= \tfrac{1}{2} \left[\ln 5 + 2 \ln x \right]$$

$$= \tfrac{1}{2} \ln 5 + \ln x$$

note: $\ln (ab)^2 = 2 \ln (ab)$

$\ln (ab^2) \neq 2 \ln (ab)$

the base-change formula

how can we calculate $\log_3 7$?

↑ note: no log-base-3 button on calculator

look at:

solve for x:

$$b^x = c$$

taking the log of both sides

$$x = \log_b c$$

$$\log b^x = \log c$$

$$x \log b = \log c$$

$$x = \frac{\log c}{\log b}$$

$$\log_b c = \frac{\log c}{\log b}$$

base-change formula:

$$\log_b M = \frac{\log_a M}{\log_a b} \quad \left(= \frac{\log M}{\log b} = \frac{\ln M}{\ln b} \right)$$

$$\log_3 7 = \frac{\log 7}{\log 3} \quad \text{or} \quad \frac{\ln 7}{\ln 3}$$

≈ 1.77124

So, calculate the following to 3 decimals:

$$\log_{22} 42 = \frac{\log 42}{\log 22} \approx 1.209$$

$$\log_{0.29} 103 = \frac{\ln 103}{\ln 0.29} \approx -3.744$$