Section 5.5: Applications

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8:49 AM

compound interest problems:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$ ← n compounding periods per year

$$A = Pe^{rt}$$ ← continuous compounding

exponential growth

$$P = P_0 e^{rt}$$

where P_0 = initial population
P = final population
r = growth rate
t = time

exponential decay

$$P = P_0 e^{-rt}$$

(note: if you use $P = P_0 e^{rt}$, will simply find that your value for r is negative)

example: Tyler deposits a certain amount of money in a term deposit at a rate of 3% per year compounded monthly. How long will it take for Tyler's money to quadruple?

$$A = P\left(1 + r\right)^{nt}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$4P = P\left(1 + \frac{0.03}{12}\right)^{12t}$$

$$\frac{4P}{P} = \frac{P}{P}(1.0025)^{12t}$$

$$4 = 1.0025^{12t}$$

$$\ln 4 = \ln 1.0025^{12t}$$

$$\frac{\ln 4}{12 \ln 1.0025} = \frac{12t \ln 1.0025}{12 \ln 1.0025}$$

$$t = \frac{\ln 4}{12 \ln 1.0025}$$

$$\approx 46.2676 \text{ years}$$

$$\approx 46 \text{ years} \quad (\text{or } 46.3 \text{ years or } 46\tfrac{1}{4} \text{ years})$$

example: According to Monday Magazine, "one rat can create 15,000 descendents in a year." Assuming that this is true,

a) calculate the doubling time for Victoria's rat population.

b) how long will it take for that one rat to have one million descendents instead?

a) $P = P_0 e^{rt}$

since this is our first word problem of this type, let's start by saying

doubling time = time to increase population by a factor of 2

$$2\cancel{P_0} = \cancel{P_0} e^{rt}$$

$$2 = e^{rt}$$

← how many variables? two! we need r to find t

so, need to find r first

$$P = P_0 e^{rt}$$

$$15000 = 1 e^{r \cdot 1}$$

$$15000 = e^{r}$$

$$\ln 15000 = r$$

$$r \approx 9.61581$$

↑ note: do <u>not</u> round here! carry at least a few extra places

going back

$$2 = e^{rt}$$

$$2 = e^{9.61581 t}$$

$$\ln 2 = 9.61581 t$$

$$\frac{\ln 2}{9.61581} = t$$

$$t = 0.072084 \text{ years} \left(\frac{365 \text{ days}}{1 \text{ year}} \right)$$

$$= 26.3 \text{ days}$$