Section 6.1: Counting Techniques

Tuesday, November 25, 2014
9:04 AM

example: How many 4-digit natural numbers are evenly divisible by 5?

method #1: 1000, 1005, 1010, ... 9995

look! it's an arithmetic sequence!
$d = 5$

$$a_n = a_1 + (n-1)d$$
$$9995 = 1000 + (n-1)5$$
$$8995 = 5(n-1)$$
$$1799 = n-1$$
$$n = 1800$$

method #2: 1000, 1005, 1010, ... 9995
5×200, 5×201, 5×202, ... 5×1999

number of terms = last − first + 1
= 1999 − 200 + 1
= 1800

method #3: 1000, 1005, 1010, ... 9995

number of choices → $\underset{1\to9}{\underline{9}}$ $\underset{0\to9}{\underline{10}}$ $\underset{0\to9}{\underline{10}}$ $\underset{0,5}{\underline{2}}$ ← now multiply

$9 \cdot 10 \cdot 10 \cdot 2 = 1800$

note: method #3 works when you are ruling at possibilites in one of the slots

however, if you are trying to find numbers divisible by 3, method #3 doesn't work

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multiplication rule:

suppose we have an event which is made up of n different independent steps

then:

$$\begin{pmatrix}\text{total number of ways the event can happen}\end{pmatrix} = \begin{pmatrix}\text{number of ways first step can happen}\end{pmatrix} \times \begin{pmatrix}\text{... second step}\end{pmatrix} \times \begin{pmatrix}\text{... third ...}\end{pmatrix} \times \cdots \begin{pmatrix}\text{last}\end{pmatrix}$$