Section 6.1: cont'd

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8:37 AM

example: How many different BC licence plates for cars are there? (Ignore personalized plates and reserved words.)

three patterns: letter - letter - letter - # - # - #

\# # # letter letter letter

letter letter # ## letter

then

total number of plates, top pattern $= \underline{26}\ \underline{26}\ \underline{26}\ \underline{10}\ \underline{10}\ \underline{10}$

$= 26^3 \cdot 10^3$

$= 17,576,000$

total # plates $= 3(17,576,000)$

$= 52,728,000$

example: In the mythical Canadian province of Gondor, licence plates follow the pattern letter letter letter number number.
Due to recent political events, the letter combination EYE is no longer allowed.
How many legal licence plates are there?

total plates = $\underline{26}\ \underline{26}\ \underline{26}\ \underline{10}\ \underline{10}$

forbidden plates = $\underline{1}\ \underline{1}\ \underline{1}\ \underline{10}\ \underline{10}$

total allowed = total − forbidden

$= 26^3 \cdot 10^2 - 10^2$

$= 1{,}757{,}500$

Tip: sometimes it's easier to calculate the number that's forbidden and subtract that from the total

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addition rule:

example: How many numbers from 1 to 20 inclusive are

a) evenly divisible by 2?

b) " " by 3?

c) " " by 2 or 3?

brute force method:

divisible by 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 $\boxed{10}$

divisible by 3: 3, 6, 9, 12, 15, 18 $\boxed{6}$

divisible by 2 or 3: 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20 note: $13 \neq \underbrace{10+6}_{\text{contains duplicates}}$

rule: $n(A \text{ or } B) = n(A) + n(B) - n(AB)$

ways
A or B
could happen

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example: How many 4-digit PINs

a) start with a 9?
b) end in a 4?
c) start with a 9 or end in a 4?
d) start with a 9 or a 4?

a) $\underline{1}\ \underline{10}\ \underline{10}\ \underline{10} = 10^3$

b) $\underline{10}\ \underline{10}\ \underline{10}\ \underline{1} = 10^3$

c) start with 9 & end in 4: $\underline{1}\ \underline{10}\ \underline{10}\ \underline{1}$

100

$n(\text{start with 9 } \underline{\text{or}} \text{ end in 4}) = n(\text{start 9}) + n(\text{end 4}) - n(\text{both})$

$= 1000 + 1000 - 100$

$= 1900$

d) $n(\text{start with 9 or 4}) = n(\text{start 9}) + n(\text{start 4}) - n(\text{both}) \to 0$

$= 1000 + 1000 - 0 = 2000$

method #2 $\underline{2}\ \underline{10}\ \underline{10}\ \underline{10}$

example: How many 5-digit, case-sensitive, alphanumeric passwords are there

a) in total?

b) that contain at least one letter and at least one number?

a) $\underline{62}\ \underline{62}\ \underline{62}\ \underline{62}\ \underline{62} = 62^5$

$= 916,132,832$

b) total allowed = total possible − forbidden

all letters: 52^5

all numbers: 10^5

total allowed $= 62^5 - 52^5 - 10^5$

$= 535,828,000$

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example: How many 4-digit PINs are there if repetition of digits is not allowed?

$\underline{10}\ \underline{9}\ \underline{8}\ \underline{7} = 5040$

note: there's a different way to calculate this

$$10 \cdot 9 \cdot 8 \cdot 7 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= \frac{10!}{6!}$$

where 10 is the number of choices per slot, 4 is the number of slots, and $(10-4)!$ is the denominator