Section 6.2: Combinations and Permutations

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9:37 AM

permutation ≡ an ordered group of r objects chosen from n possibilities without repetition

example: 4 digit PIN without repetition → 4 digits in order from 10 possibilities

notation:

$$_nP_r = \frac{n!}{(n-r)!}$$

other commonly used symbols: P_r^n , $P(n,r)$

example: How many 5-digit case-sensitive alphanumeric passwords are there if repetition is not allowed?

$n = 62$
$r = 5$

$_{62}P_5 = 776\ 520\ 240$

is repetition allowed? no → either permutation combination

does order matter? yes → permutation
no → combination

example: how many ways are there to order a group

of 5 objects?

$5 \quad 4 \quad 3 \quad 2 \quad 1 = 5! = 120$

or $\quad {}_5P_5 = 120$

note: ${}_nP_r = \dfrac{n!}{(n-r)!}$

${}_5P_5 = \dfrac{5!}{0!} = 5!$

so, there are $r!$ ways to arrange r objects

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combination: an <u>unordered</u> group of r objects chosen <u>without repetition</u> from n possibilities

$${}_nC_r = \frac{n!}{r!\,(n-r)!} \qquad \left[= \frac{{}_nP_r}{r!} \right]$$

note: other commonly used notations:

$$C_r^n, \quad C(n,r), \quad \binom{n}{r}$$

example: Pat is ordering pizza from ~~Pesago~~ <insert pizza place of choice>. she has the choice of 3 toppings* chosen from a list of 15. How many different pizzas could she potentially order?

* no repetition allowed

$_{15}C_3 = 455$

what if, in addition to 3 toppings chosen without repetition from a list of 15, you could also choose one out of 3 different crusts?

ways of selecting toppings : $_{15}C_3$

ways of picking crust : $_3C_1$

pizzas : $_{15}C_3 \cdot {}_3C_1$

$= 455 \cdot 3$

$= 1365$

In the BC 6/49 lottery, the customer chooses without repetition six numbers from 49 possibilities. These numbers are then compared with the six numbers drawn at random at the end of the week.

How many different choices could you make that give you exactly three winning numbers? Four winning numbers?

three winning numbers :

$_6C_3 \cdot {}_{43}C_3$

6 winning numbers
43 losing numbers

6 picked at end of week — you have 3 that match

$${}_6C_3 \cdot {}_{43}C_3 = 20 \cdot 12341 = 246820$$

four winning numbers

$${}_6C_4 \; {}_{43}C_2 = 15 \cdot 903 = 13545$$

So, what's the total number of choices you could potentially make?

$${}_{49}C_6 = 13,983,816$$

so, what's your chance of winning the jackpot?

$$\frac{1}{13,983,816}$$