Math 163 – Test #1: Form A

October 23, 2015 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 40 points

1. Let A = {1, 3, 5, 7}, B = {1, 3, 5, 7, ...}, and U = {x | x is an odd integer}. State whether the following are T (true) or F (false). (6 points)

   a) $A \cup B = A$ $A \cup B = \{1, 3, 5, 7, \ldots\}$ F

   b) $B = U$ ↖ contains negative numbers F

   c) $A \subseteq A$ T

   d) $\varnothing \subset B$ T

   e) $\overline{A} = \{9, 11, 13, \ldots\} \in \bar{A}$ has negative numbers F

   f) $\{-1\} \in U$ F

   ↑ needs to be $\subseteq$ or $\subset$ for this to be true

2. Let p denote "John took the bus to work" and q denote "John walked home." Rewrite the following English sentences in terms of logical symbols (i.e. $p \wedge q$, $p \vee q$). Do not simplify! (4 points)

   a) John walked home or he didn't take the bus to work. $q \vee \bar{p}$

   b) Either John didn't take the bus to work or he didn't walk home but not both. $\bar{p} \oplus \bar{q}$

   c) It is not true that John both took the bus to work and walked home. $\overline{p \wedge q}$

   d) John walked home or he didn't walk home. $q \vee \bar{q}$ } either
   $q \oplus \bar{q}$

note: if mixed up $\wedge$ and $\vee$ for entire question, take off max of (-2)

but (-1) if used p

3. For the pair of sentences below, is the second the negation of the first? (2 points)

   a) The number of keys on Pat's keyring is positive. Pat has no keys on her keyring.
   can't have negative keys! Yes

   b) Many students were late for class. No students were late for class. No

4. Consider the sets $U=\{1,2,3,4,5,6\}$, $A=\{1,4\}$, and $B=\{4,5,6\}$. Each part is only worth one point, so you don't need to show any work. (4 points)

a) Find $\overline{B}$. $\{1,2,3\}$

(-½) for single missing/extra element

b) Find $\overline{A}\cup\overline{B}$. $= \{2,3,5,6\} \cup \{1,2,3\}$ $\{1,2,3,5,6\}$

c) Find $\overline{A\cap\overline{B}}$. $A \cap \overline{B} = \{1,4\} \cap \{1,2,3\} = \{1\}$ $\{2,3,4,5,6\}$

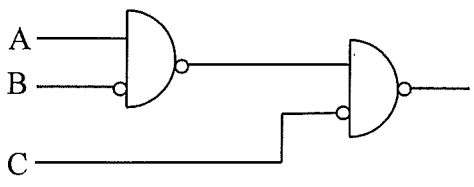
d) Find $B\cap\varnothing$. $\varnothing$

5. Answer the following questions with "Yes", "No", or "Maybe". (2 points)

a) Larry blogs daily or answers his email. Does he blog daily? Maybe

b) Frank ordered soup and salad. Did he order soup or salad? Yes

6. Write the Boolean expression that corresponds to the following gate diagram. Do not simplify! (3 points)



(-2) really screwing up the negations

$\overline{A\overline{B}\,\overline{C}}$

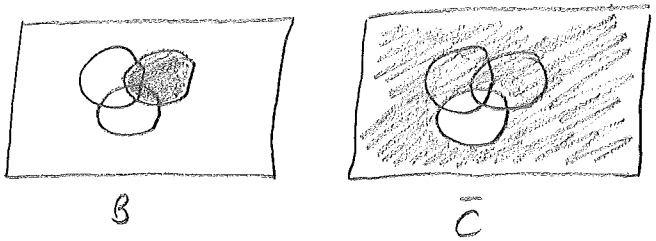
(-1½) for using "or", not "and"
(-1) for not using Boolean
(-1) each mistake

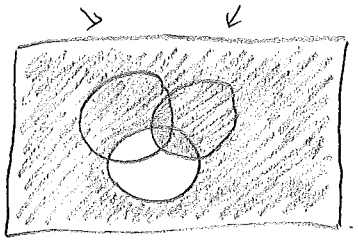
7. List all of the proper subsets of the set {1,2}. (2 points)

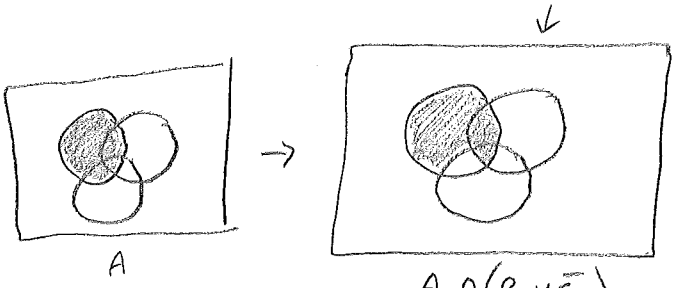
$\varnothing, \{1\}, \{2\}$

(½) each for $\varnothing$, $\{1\}$, $\{2\}$
(½) for not including $\{1,2\}$
(-½) for any other sets, each

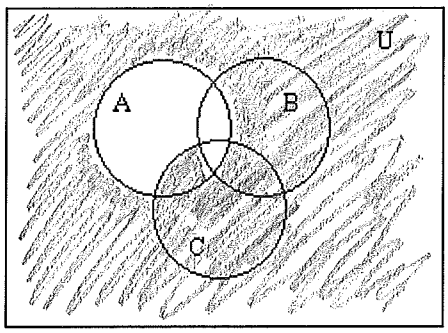
8. Represent $\overline{A \cap (B \cup \bar{C})}$ on the following Venn diagram by shading in the appropriate regions. Show intermediate steps on separate sketches to get full credit. (5 points)







(-1) each mistake for max of (-2) per diagram



$A \cap (B \cup \bar{C})$

9. Consider the sets $U = \{1,2,3,...10\}$, $A = \{1,2,3,6,9\}$, and $B = \{2,4,6,8,9\}$. Use the **computer representation of sets** to find the following. To get full credit, you must show your work. (5 points)

$\overline{\bar{A} \cap \bar{B}} \cup A$

	1234	5678	910
A	1110	0100	10
B	0101	0101	10
$\bar{A}$	0001	1011	01
$\bar{B}$	1010	1010	01
$\bar{A} \cap \bar{B}$	0000	1010	01
$\overline{\bar{A} \cap \bar{B}}$	1111	0101	10
$\overline{\bar{A} \cap \bar{B}} \cup A$	1111	0101	10

(-1) if final answer not in set notation

← (-2) if didn't show work and many errors

$\overline{\bar{A} \cap \bar{B}} \cup A = \{1,2,3,4,6,8,9\}$

For the questions on this page: if you are using the Laws of Logic, remember to use one law of logic per line, and be sure to state the name of the law you are using!

10. Prove that the following Boolean expressions are equivalent by using the laws of logic. If you're stuck, try using a truth table for part marks. (5 points)

$$A+\overline{A+B}=A+\overline{B}C+\overline{B}\,\overline{C}$$

$A+\bar{A}\bar{B}=$ " De Morgans

$A+\bar{B} = A+\bar{B}C+\bar{B}\bar{C}$ absorption

$= A+\bar{B}(C+\bar{C})$ distributive

$= A+\bar{B}\cdot 1$ complement

$= A+\bar{B}$ identity

✓

-1 each minor mistake

-2 major error

-½ name of law wrong

11. Simplify the following. This is the nasty question I promised you and credit will only be awarded if the laws of logic are used to simplify the expression. (2 points)

$$\overline{\overline{p\vee q}\vee\overline{q\vee p}}$$

$(\bar{p}\vee\bar{q})\wedge(\bar{q}\vee p)$ De Morgan's

$\bar{q}\vee(\bar{p}\wedge p)$ distributive

$\bar{q}\vee 0$ complement

$\bar{q}$ identity

-½ if omitted brackets but rest of steps okay

-1 if omitted brackets and immediately made an error