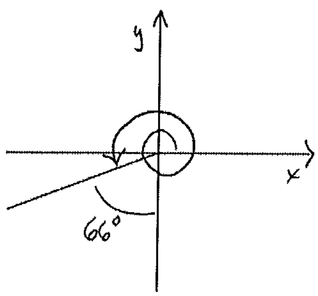
(white version)

Math 163 – Test #3 (Version A)

December 4, 2015 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 40 points

1. Consider the following sketch of an angle θ in standard position. The swirly line indicates the number and direction of rotations. Calculate the size of the angle, and list one positive and one negative coterminal angle. Also, state the reference angle and whether tan θ is positive or negative. (5 points)



angle θ: $564°$

coterminal angles: $204°$, $-156°$ (among many other possibilities)

reference angle: $24°$

tan θ: positive / negative

other popular coterminal: $924°$, $-516°$

2. Use a calculator to evaluate the following trig functions. Round any approximate answers to two decimal places. (4 points)

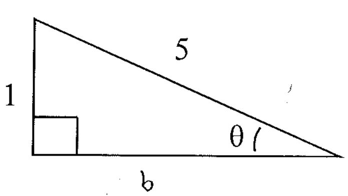
a) $\sin(81.5°) \approx 0.989016$ — 0.99

b) $\cos(-1022°) \approx 0.529919$ — 0.53

c) $\tan^{-1}(1.23) \approx 50.8886°$ (or 0.888174 rads) — $50.89°$

d) $\cos^{-1}(1.23)$ — undefined (or DNE)

3. Find the **exact** values of the three basic trig functions of θ for the following triangle. (5 points)



$a^2 + b^2 = c^2$

$b^2 = c^2 - a^2$

$= 25 - 1 = 24$

$b = \sqrt{24} = 2\sqrt{6}$ (1)

$\sin\theta = \frac{opp}{hyp} = \frac{1}{5}$ (1)

$\cos\theta = \frac{adj}{hyp} = \frac{2\sqrt{6}}{5}$ (1)

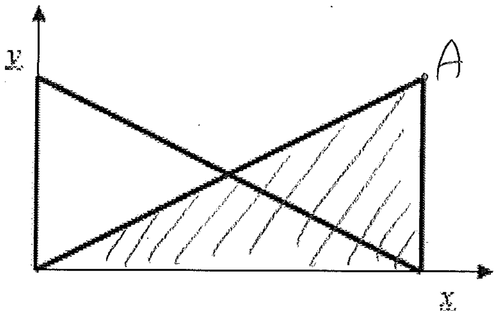
$\tan\theta = \frac{opp}{adj} = \frac{1}{2\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right) = \frac{\sqrt{6}}{12}$ (1)

b not simplified ($-\frac{1}{2}$)

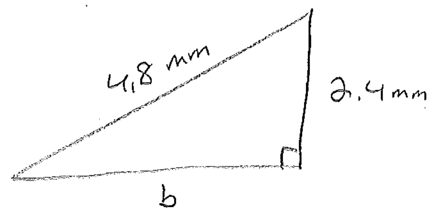
tan θ not simplified ($-\frac{1}{2}$)

not exact values (-2)

4. The JOIN operator is a symmetrical symbol made from two equilateral triangles, as shown in the diagram below. If each equilateral triangle has a side of length 2.4 mm, what are the coordinates of the point at the upper right hand corner of the diagram? (Be sure to include your units and round appropriately.) (4 points)



shaded triangle is:



$a^2 + b^2 = c^2$

$b^2 = c^2 - a^2$

$= (4.8)^2 - (2.4)^2$

$= 17.28$

$b = 4.15692$

$= 4.2$ mm

The coords of point A are (4.2mm, 2.4 mm)

5. Sketch the graph of the following function. Include at least two accurate points on the graph, be sure to label your graph appropriately, and indicate the location of any asymptotes. (5 points)

$$y = \left(\frac{1}{3}\right)^x$$

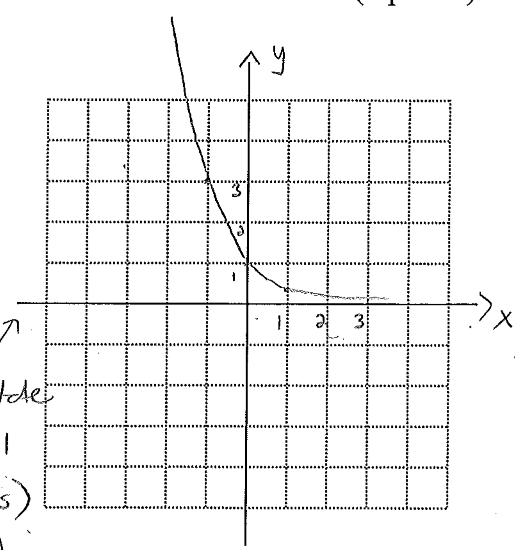
two accurate points 1

correct end behavior 2

labels 1

LHS linear -½

crosses x-axis -1



asymptote is y = 1 (x-axis)

1

6. Solve the following equations. Give exact answers. (4 points)

a) $3^{2x} = 22$

$x = \frac{1}{2}\log_3 22$

$2x = \log_3 22$

$x = \frac{1}{2}\log_3 22$ or $\frac{1}{2}\frac{\log 22}{\log 3}$ or $\frac{1}{2}\frac{\ln 22}{\ln 3}$

(-1) approx answer

(-1/2) both exact & approx with no info as to which is final

b) $\log_2(23-3y) = \log_2(2y-12)$

$y = 7$

$23 - 3y = 2y - 12$

$35 = 5y$

$y = 7$

7. Solve the following equation. Give an **approximate** solution rounded to two decimal places. (4 points)

$3(1.01)^{x+1} = 24$

$x = 207.98$

$1.01^{x+1} = 8$

$\ln 1.01^{x+1} = \ln 8$

$(x+1)\ln 1.01 = \ln 8$

$x + 1 = \frac{\ln 8}{\ln 1.01}$

$x = \frac{\ln 8}{\ln 1.01} - 1$

≈ 207.982

≈ 207.98

or

$1.01^{x+1} = 8$

$x + 1 = \log_{1.01} 8$

$x = \log_{1.01} 8 - 1$

$= \frac{\ln 8}{\ln 1.01} - 1$

etc

didn't divide by 3 first (-2) and then brought (x+1) out front

8. Write as a single logarithm and simplify. Show your work. (3 points)

$\log_b\left(ab^2\right) - \log_b a$ $\underline{\quad 2 \quad}$

$\log_b \dfrac{ab^2}{a}$

$\log_b b^2$

2

9. Computer scientists studying "linkrot" have found that exponential decay is a good model for the number of links that remain usable on a web page as a function of time. Two researchers created a page with 510 links to science education resources in August 2000. Exactly two years later, they found that only 370 of those links were still working. If their model is accurate, how long will it take (from the original date of August 2000) for only half of the original links to still work? (6 points)

finding r:

$P = P_0 e^{-rt}$

510 links → 370 links in 2 years

$370 = 510\, e^{-r\cdot 2}$

$\dfrac{370}{510} = e^{-2r}$

$\ln\left(\dfrac{37}{51}\right) = -2r$

$r = -\frac{1}{2}\ln\left(\frac{37}{51}\right)$

≈ 0.160454

now find "half life":

$P = P_0 e^{-rt}$

$P_0 \rightarrow \frac{1}{2} P_0$ in ? years

$\frac{1}{2} P_0 = P_0 e^{-0.160454t}$

$\frac{1}{2} = e^{-0.160454t}$

$\ln \frac{1}{2} = -0.160454\, t$

$t = -\dfrac{\ln \frac{1}{2}}{0.160454}$

≈ 4.31992 years

≈ 4.3 years

(4 years and 117 days if you insist)