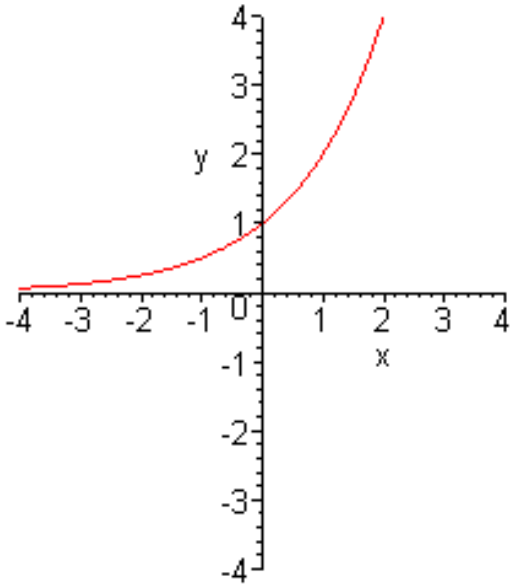
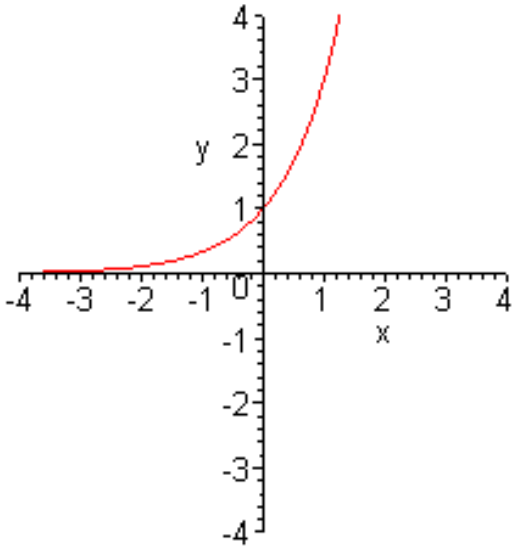
Section 5.1: Exponential Functions

Solutions to all questions

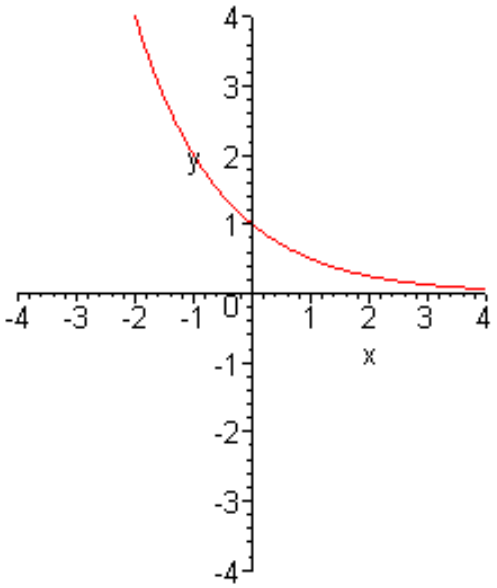
1. $y = 2^x$



2. $y = 3^x$

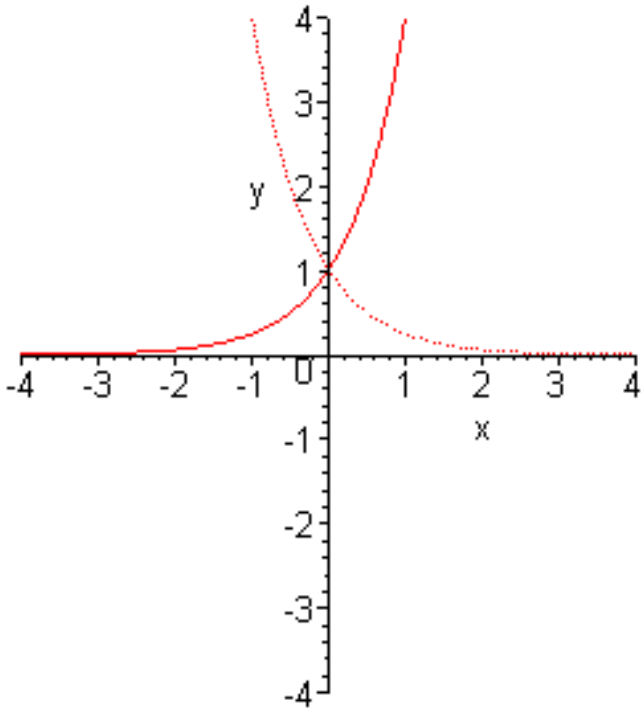


3. $y = \left(\frac{1}{2}\right)^x$



4. $y = 2^{-x}$ – same as graph for question 3, since $y = 2^{-x} = \left(2^{-1}\right)^x = \left(\frac{1}{2}\right)^x$

5. $y = 4^x$ is the solid line and $y = \left(\frac{1}{4}\right)^x$ is the dotted line



6. $3^{x+2} = 9$
$3^{x+2} = 3^2$
$x + 2 = 2$
$x = 0$

7. $6^x = \frac{1}{36}$
$6^x = 6^{-2}$
$x = -2$

8. $10^{-x} = 0.01$
$10^{-x} = 10^{-2}$
$-x = -2$
$x = 2$

9. $4^{5-x} = 64$
$4^{5-x} = 4^3$
$5 - x = 3$
$2 = x$
$x = 2$

10. $8^x = \frac{1}{2}$
$\left(2^3\right)^x = 2^{-1}$
$2^{3x} = 2^{-1}$
$3x = -1$
$x = -\frac{1}{3}$

11. $5^{2x} = 125$
$5^{2x} = 5^3$
$2x = 3$
$x = \frac{3}{2}$

12. $2^{5+x} = 256$
$2^{5+x} = 2^8$
$5 + x = 8$
$x = 3$

13. $64^{5+x} = 4$
$\left(4^3\right)^{5+x} = 4^1$
$4^{3(5+x)} = 4^1$
$3(5+x) = 1$
$15 + 3x = 1$
$3x = -14$
$x = -\frac{14}{3}$

14. $100^{5-x} = 1000^2$
$\left(10^2\right)^{5-x} = \left(10^3\right)^2$
$10^{2(5-x)} = 10^6$
$2(5-x) = 6$
$10 - 2x = 6$
$4 = 2x$
$x = 2$

15. $49^{2m} = 7^{m+1}$
$7^{4m} = 7^{m+1}$
$4m = m + 1$
$3m = 1$
$m = \frac{1}{3}$

16. $2^{b+1} = 8^{1-b}$
$2^{b+1} = 2^{3(1-b)}$
$b + 1 = 3(1-b)$
$b + 1 = 3 - 3b$
$4b = 2$
$b = \frac{1}{2}$

17. $4^x = \sqrt{2}$
$2^{2x} = 2^{1/2}$
$2x = 1/2$
$x = 1/4$

18. $\left(\sqrt{2}\right)^y = 4$
$2^{y/2} = 2^2$
$y/2 = 2$
$y = 4$

19. $\left(\sqrt{2}\right)^k = \frac{1}{2}$
$2^{k/2} = 2^{-1}$
$k/2 = -1$
$k = -2$

20. $0.1^x = 100$
$10^{-x} = 10^2$
$-x = 2$
$x = -2$

21. $0.5^{0.5x} = 16$
$(1/2)^{x/2} = 2^4$
$2^{-x/2} = 2^4$
$-x/2 = 4$
$x = -8$

22. $5^{2x} = 5^{3x}$
$2x = 3x$
$0 = 3x - 2x$
$0 = x$
$x = 0$

23. $$\begin{aligned} A &= P\left(1+\frac{r}{n}\right)^{nt} \\ &= 5000\left(1+\frac{0.05}{2}\right)^{2\times 5} \\ &= 6400.42 \end{aligned}$$ So Nicole will have $6400.42 in her account after 5 years.

24. $$\begin{aligned} A &= Pe^{rt} \\ &= 800e^{0.03\times 6} \\ &= 957.77 \end{aligned}$$ Peter will have $957.77 after six years.

25. $$\begin{aligned} A &= P\left(1+\frac{r}{n}\right)^{nt} \\ &= 2000\left(1+\frac{0.08}{365}\right)^{365\times 1} \\ &= 2166.56 \end{aligned}$$ Darcy owes the bank $2166.56 after one year. (Note: you will still get the same answer if you use 365.25 days in a year. And now that I've reread the question, wouldn't he owe the bank zero dollars if he's already paid the loan off?)

26. $$\begin{aligned} A &= P\left(1+\frac{r}{n}\right)^{nt} \\ &= 10000\left(1+\frac{0.12}{1}\right)^{10} \\ &= 31058.48 \end{aligned}$$ The investment will be worth $31 058.48 after ten years.

27. For parts a, b, and c, the equation will be $$\begin{aligned} A &= P\left(1+\frac{r}{n}\right)^{nt} \\ &= 1500\left(1+\frac{0.02}{n}\right)^{3n} \end{aligned}$$.

a) $n = 1$, so $A = 1591.81$ David will have $1591.81 after three years.
b) $n = 52$, so $A = 1592.74$ David will have $1592.74 after three years.
c) $n = 365$, so $A = 1592.75$ David will have $1592.75 after three years.
d) Compounding continuously requires the equation $$\begin{aligned} A &= Pe^{rt} \\ &= 1500e^{0.02\times 3} \\ &= 1592.75 \end{aligned}$$, so David will have $1592.75 after three years (no significant difference between this and daily compounding).