

Section 5.3: Properties of Logarithms

Solutions

1. $\log(M + N) = \log M + \log N$ – False
2. $\log(MN) = (\log M)(\log N)$ – False
3. $\log(MN) = \log M + \log N$ – True (this is the product rule)
4. $n \log x = \log(nx)$ – False
5. $\frac{\log M}{\log N} = \log\left(\frac{M}{N}\right)$ – False
6. $\frac{\log M}{\log N} = \frac{M}{N}$ (logs cancel) – False
7. $\frac{\log M}{\log N} = \log_N M$ – True (this is the base-change formula)
8. $n \log x = \log(x^n)$ – True (this is the power rule)
9. $\log 2000 - \log 2 = \log \frac{2000}{2} = \log 1000 = 3$
10. $\log 2 + \log 5 = \log 10 = 1$
11. $\log_2 4x - \log_2 x = \log_2 \frac{4x}{x} = \log_2 4 = 2$
12. $\log_3 5 + \log_3 2 = \log_3 10$ (note: doesn't simplify further)
13. $\log_5 x - \log_5 y + \log_5 z = \log_5 \left(\frac{xz}{y}\right)$
14. $\log_a a^3 - 2 \log_a a = 3 \log_a a - 2 \log_a a = \log_a a = 1$
15. $\log 2x = \log 2 + \log x$
16. $\log\left(\frac{x}{4}\right) = \log x - \log 4 = \log x - \log 2^2 = \log x - 2 \log 2$

$$17. \log(8x^3) = \log 8 + \log x^3 = \log 2^3 + 3\log x = 3\log 2 + 3\log x$$

$$18. \log \sqrt{x} = \log x^{1/2} = \frac{1}{2}\log x$$

$$19. \log\left(\frac{1}{x}\right) = \log x^{-1} = -\log x$$

$$20. \log(2x)^7 = 7\log(2x) = 7(\log 2 + \log x) = 7\log 2 + 7\log x$$

$$21. 2\log x - 3\log y = \log x^2 - \log y^3 = \log\left(\frac{x^2}{y^3}\right)$$

$$22. \frac{1}{3}\log x + 5\log 2 = \log x^{1/3} + \log 2^5 = \log(32x^{1/3})$$

$$23. 2\log 3 - 3\log y = \log 3^2 - \log y^3 = \log\left(\frac{9}{y^3}\right)$$

$$24. \log 5 + 3\log 2 - \log 4 = \log 5 + \log 2^3 - \log 4 = \log \frac{5 \cdot 8}{4} = \log 10 = 1$$

$$25. \frac{1}{2}\log 4 + \frac{1}{3}\log 27 = \log 4^{1/2} + \log 27^{1/3} = \log 2 + \log 3 = \log 6$$

$$26. 2\log x + 3\log x^2 = 2\log x + 6\log x = 8\log x \quad (\text{or } \log x^8)$$

$$27. \log_7 10 = \frac{\ln 10}{\ln 7} = 1.183 \quad (\text{note that you could also do a ratio of log rather than ln})$$

$$28. \log_{0.2} 15 = \frac{\ln 15}{\ln 0.2} = -1.683$$

$$29. \log_{1.05} 2 = \frac{\ln 2}{\ln 1.05} = 14.207$$

$$30. \log_2 1.05 = \frac{\ln 1.05}{\ln 2} = 0.0703$$

$$31. \log_{500} 1000 = \frac{\ln 1000}{\ln 500} = 1.112$$

$$32. \log_{0.001} 10 = \frac{\ln 10}{\ln 0.001} = -0.333$$