

## Section 5.4: Solving Equations with Exponents and Logs

### Solutions

1. 
$$5^x = 0.1$$
$$x = \log_5 0.1 = \frac{\ln 0.1}{\ln 5} = -1.431$$

2. 
$$3^{2x} = 24$$
$$2x = \log_3 24$$
$$x = \frac{1}{2} \log_3 24 = \frac{\ln 24}{2 \ln 3} = 1.446$$

3. 
$$10^q = 15$$
$$q = \log 15 = 1.176$$

4. 
$$2^{1-x} = 0.00004$$
$$1-x = \log_2 0.00004$$
$$x = 1 - \log_2 0.00004 = 1 - \frac{\ln 0.00004}{\ln 2} = 15.610$$

5. 
$$6^{2x+1} = 36$$
$$6^{2x+1} = 6^2$$
$$2x+1 = 2$$
$$2x = 1$$
$$x = 1/2 = 0.5$$

6. 
$$7^{5-m} = 10$$
$$5-m = \log_7 10$$
$$m = 5 - \log_7 10 = 5 - \frac{\ln 10}{\ln 7} = 3.817$$

7. 
$$3^{x-6} = 1$$
$$3^{x-6} = 3^0$$
$$x-6 = 0$$
$$x = 6$$

8.  $25 = 0.5^y$   
 $\log_{0.5} 25 = y$   
 $y = \log_{0.5} 25 = \frac{\ln 25}{\ln 0.5} = -4.644$

9.  $1.02^x = 3$   
 $x = \log_{1.02} 3 = \frac{\ln 3}{\ln 1.02} = 55.478$

10.  $e^{-5t} = 0.25$   
 $-5t = \ln 0.25$   
 $t = -\frac{\ln 0.25}{5} = 0.277$

11.  $e^{7t} = 3$   
 $7t = \ln 3$   
 $t = \frac{\ln 3}{7} = 0.157$

12.  $1.01^{10x} = 5$   
 $10x = \log_{1.01} 5$   
 $x = \frac{\log_{1.01} 5}{10} = \frac{\ln 5}{10 \ln 1.01} = 16.175$

13.  $5(1.015)^{8x} = 10$   
 $(1.015)^{8x} = 2$   
 $8x = \log_{1.015} 2$   
 $x = \frac{\log_{1.015} 2}{8} = \frac{\ln 2}{8 \ln 1.015} = 5.819$

14.  $10e^{-5t} = 2$   
 $e^{-5t} = \frac{1}{5}$   
 $-5t = \ln(1/5)$   
 $t = -\frac{\ln(1/5)}{5} = 0.321$

$$\begin{aligned}
 15. \quad & 2(1.02)^{3t} = 8 \\
 & (1.02)^{3t} = 4 \\
 & 3t = \log_{1.02} 4 \\
 & t = \frac{\log_{1.02} 4}{3} = \frac{\ln 4}{3 \ln 1.02} = 23.335
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & 1000e^{-10t} = 100 \\
 & e^{-10t} = \frac{1}{10} \\
 & -10t = \ln(1/10) \\
 & t = -\frac{\ln(1/10)}{10} = 0.230
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \log(1-x) = 1 \\
 & 1-x = 10^1 \\
 & -x = 9 \\
 & x = -9
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \log(2x+1) = 0 \\
 & 2x+1 = 10^0 \\
 & 2x+1 = 1 \\
 & 2x = 0 \\
 & x = 0
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \log_2(x-5) = 3 \\
 & x-5 = 2^3 \\
 & x-5 = 8 \\
 & x = 13
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \ln x = 2 \\
 & x = e^2
 \end{aligned}$$

21.  $\log_x(3) = -1$   
 $3 = x^{-1}$   
 $3 = \frac{1}{x}$   
 $x = \frac{1}{3}$

22.  $\log_x(8) = 3$   
 $8 = x^3$   
 $x = 2$

23.  $\log_3(x-2) = -1$   
 $x-2 = 3^{-1}$   
 $x = 2 + \frac{1}{3}$   
 $x = \frac{7}{3}$

24.  $\log_5(x^3 - 2) = 2$   
 $x^3 - 2 = 5^2$   
 $x^3 = 27$   
 $x = 3$

25.  $\log_x(8) = 1/2$   
 $8 = x^{1/2}$   
 $x = 64$

26.  $\log_{x+2}(64) = 3$   
 $64 = (x+2)^3$   
 $4 = x+2$   
 $x = 2$

27.  $\log(x+5) = \log(7)$   
 $x+5 = 7$   
 $x = 2$

28.  $\ln(1-x) = \ln(9+x)$   
 $1-x = 9+x$   
 $-2x = 8$   
 $x = -4$

29.  $\log_2(x^3 - 1) = \log_2(26)$   
 $x^3 - 1 = 26$   
 $x^3 = 27$   
 $x = 3$

30.  $\log_{1.02}(x^2 + 5) = \log_{1.02}(x^2 + x)$   
 $x^2 + 5 = x^2 + x$   
 $5 = x$   
 $x = 5$

31.  $\ln\left(\frac{2}{3}x + 1\right) = \ln\left(\frac{3}{2}x - 4\right)$   
 $\frac{2}{3}x + 1 = \frac{3}{2}x - 4$  (multiply both sides by 6)  
 $4x + 6 = 9x - 2$  4  
 $30 = 5x$   
 $x = 6$

32.  $\log(0.01x) = \log(5)$   
 $0.01x = 5$   
 $x = 500$