Section 5.5: Applications

Solutions

1.
$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$700 = 500\left(1+\frac{0.08}{4}\right)^{4t}$$
$$\frac{7}{5} = (1.02)^{4t}$$
$$\ln\left(\frac{7}{5}\right) = \ln(1.02)^{4t}$$
$$\ln\left(\frac{7}{5}\right) = 4t\ln 1.02$$
$$t = \frac{\ln(7/5)}{4\ln 1.02} = 4.2478$$

The account will contain \$700 in $4\frac{1}{4}$ years.

2.
$$A = Pe^{rt}$$
$$2A_0 = Pe^{0.05t}$$
$$2 = e^{0.05t}$$
$$\ln 2 = 0.05t$$
$$t = \frac{\ln 2}{0.05} = 13.8629$$

The investment will double in 13.9 years. (Okay to round to 14.)

3.

solution #1:
$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$2P = P\left(1+\frac{0.12}{2}\right)^{2\times t}$$
$$2 = (1.06)^{2t}$$
$$\log_{1.06} 2 = 2t$$
$$t = \frac{\log_{1.06} 2}{2} = \frac{\ln 2}{2\ln 1.06} = 5.94783$$

solution #2:
$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$2P = P\left(1+\frac{0.12}{2}\right)^{2\times t}$$
$$2 = (1.06)^{2t}$$
$$\ln 2 = \ln(1.06)^{2t}$$
$$\ln 2 = 2t\ln 1.06$$
$$t = \frac{\ln 2}{2\ln 1.06} = 5.94783$$

Either solution gives the same answer: Charlotte's investment will double in just under 6 years.

4. $$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$2500 = P\left(1+\frac{0.12}{12}\right)^{12\times 5}$$
$$2500 = P(1.01)^{60}$$
$$P = \frac{2500}{1.01^{60}} = 1376.12$$
Doug invested $1376.12 initially.

5. $$A = Pe^{rt}$$
$$727.50 = 500e^{r5}$$
$$1.455 = e^{5r}$$
$$\ln 1.455 = 5r$$
$$r = \frac{\ln 1.455}{5} = 0.075001$$
The interest rate on Steve's account is 7.5%.

6. a) monthly:
$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$5\ 800\ 000 = 1600\left(1+\frac{0.06}{12}\right)^{12t}$$
$$3625 = (1.005)^{12t}$$
$$\ln 3625 = \ln(1.005)^{12t}$$
$$\ln 3625 = 12t \ln 1.005$$
$$t = \frac{\ln 3625}{12 \ln 1.005} = 136.935$$

b) continuously:
$$A = Pe^{rt}$$
$$5,800,000 = 1600e^{0.06t}$$
$$3625 = e^{0.06t}$$
$$\ln 3625 = 0.06t$$
$$t = \frac{\ln 3625}{0.06} = 136.593$$

So, in either case, Pat was cryogenically frozen for 137 years.

7. For this problem, you are starting with 0.05 of the population and after two years, getting the full 100% or just 1 times the population back.

find r:

$$\begin{aligned}
&P = P_o e^{rt} \\
&P = 0.05Pe^{r(2)} \\
&20 = e^{2r} \\
&\ln(20) = 2r \\
&r = \frac{\ln(20)}{2} \\
&r = 1.49787
\end{aligned}$$

find t:

$$\begin{aligned}
&P = P_o e^{rt} \\
&2P_o = P_o e^{1.49787t} \\
&2 = e^{1.49787t} \\
&\ln(2) = 1.49787t \\
&t = \frac{\ln(2)}{1.49787} \\
&t = 0.462756
\end{aligned}$$

The doubling time for fluffy bunnies at UVic is 0.46 years (just over 5½ months).

8. If each vampire creates another vampire during the full moon, then the doubling time for the vampire population is one month. You can either do this problem in months, if you wish, remembering that your final answer will also be in months, or you can choose another unit providing that you are consistent. I will choose years for my answer, assuming 12 full moons per year (not totally accurate, but will get approximately the same answer):

find r:

$$\begin{aligned}
&P = P_o e^{rt} \\
&2P_o = P_o e^{r\left(\frac{1}{12}\right)} \\
&2 = e^{r/12} \\
&\ln(2) = \frac{r}{12} \\
&r = 12\ln(2) \\
&r = 8.31777
\end{aligned}$$

find t:

$$\begin{aligned}
&P = P_o e^{8.31777t} \\
&2{,}400{,}000 = 1e^{8.31777t} \\
&2{,}400{,}000 = e^{8.31777t} \\
&\ln(2{,}400{,}000) = 8.31777t \\
&t = \frac{\ln(2{,}400{,}000)}{8.31777} \\
&t = 1.76622
\end{aligned}$$

It would take 1.77 years (just over 1 ¾ years) to turn everyone into a vampire in Transylvania. I would have to conclude that either there were no vampires in Transylvania or their vampire hunters were very good indeed!

9. For this problem, I will take my initial P_o as 1000. Then my time will be measured from 1999, so for P = 10,000 in 2004 my t will be 5 years.

find r:

$$\begin{aligned}
&P = P_o e^{rt} \\
&10{,}000 = 1000e^{r5} \\
&10 = e^{5r} \\
&\ln(10) = 5r \\
&r = \frac{\ln(10)}{5} \\
&r = 0.460517
\end{aligned}$$

find t:

$$\begin{aligned}
&P = P_o e^{rt} \\
&5000 = 1000e^{0.460517t} \\
&5 = e^{0.460517t} \\
&\ln(5) = 0.460517t \\
&t = \frac{\ln(5)}{0.460517} \\
&t = 3.49485
\end{aligned}$$

Then you could get 5000 pixels 3.5 years after the initial time, so somewhere in 2002 to 2003. (The point is that it's not **halfway** in time between 1999 and 2004.)

10. a) In one year, $\boxed{\begin{aligned} &P = P_o e^{rt} \\ &P = 6.070 \times 10^9 e^{0.0114(1)} \\ &P = 6.13959 \times 10^9 \end{aligned}}$ so the change in population is (6.140 – 6.070) billion people = 0.07 billion people = 70 million people. In one day, $\boxed{\begin{aligned} &P = P_o e^{rt} \\ &P = 6.070 \times 10^9 e^{0.0114(1/365)} \\ &P = 6.07019 \times 10^9 \end{aligned}}$ so the change in population is 0.00019 billion people, or 0.19 million people, or 190,000 people in one day. That's a lot of people!

b) $\boxed{\begin{aligned} &P = P_o e^{rt} \\ &2P_o = P_o e^{0.0114t} \\ &2 = e^{0.0114t} \\ &\ln(2) = 0.0114t \\ &t = \frac{\ln(2)}{0.0114} \\ &r = 60.8 \end{aligned}}$ The doubling time is 60.8 years.

c) $\boxed{\begin{aligned} &P = P_o e^{rt} \\ &P = 6.070 \times 10^9 e^{0.0114(50)} \\ &P = 1.0733 \times 10^{10} \end{aligned}}$ There will be 10.73 billion people in 2050, assuming that the rate of growth remains constant.

11. find r: $\boxed{\begin{aligned} &P = P_o e^{-rt} \\ &\frac{1}{2} P_o = P_o e^{-r(3.5)} \\ &\frac{1}{2} = e^{-3.5r} \\ &\ln\left(\frac{1}{2}\right) = -3.5r \\ &r = \frac{\ln(1/2)}{-3.5} = 0.198042 \end{aligned}}$ find A: $\boxed{\begin{aligned} &P = P_o e^{-rt} \\ &P = 1e^{-0.198042(1)} \\ &P = 0.820335 \end{aligned}}$ There will be 0.82 g left.

12. find r:

$$\begin{aligned} P &= P_o e^{-rt} \\ 0.35P_o &= P_o e^{-r(5)} \\ 0.35 &= e^{-5r} \\ \ln(0.35) &= -5r \\ r &= \frac{\ln(0.35)}{-5} \\ r &= 0.209964 \end{aligned}$$

find t:

$$\begin{aligned} P &= P_o e^{-rt} \\ \frac{1}{2}P_o &= P_o e^{-0.209964t} \\ \frac{1}{2} &= e^{-0.209964t} \\ \ln\left(\frac{1}{2}\right) &= -0.209964t \\ t &= \frac{\ln(1/2)}{-0.209964} \\ t &= 3.30126 \end{aligned}$$

The halflife is 3.3 days.

13. I'm going to take my unit as millions of years.

find r:

$$\begin{aligned} P &= P_o e^{-rt} \\ 0.8A_o &= P_o e^{-r(250)} \\ 0.8 &= e^{-250r} \\ \ln(0.8) &= -250r \\ r &= \frac{\ln(0.8)}{-250} \\ r &= 0.000892574 \end{aligned}$$

find A:

$$\begin{aligned} P &= P_o e^{-rt} \\ P &= P_o e^{-0.000892574(600)} \\ P &= P_o(0.585) \\ P &= 0.585P_o \end{aligned}$$

find t:

$$\begin{aligned} P &= P_o e^{-rt} \\ \frac{1}{2}P_o &= P_o e^{-0.000892574t} \\ \frac{1}{2} &= e^{-0.000892574t} \\ \ln\left(\frac{1}{2}\right) &= -0.000892574t \\ t &= \frac{\ln(1/2)}{-0.000892574} \\ t &= 776.57 \end{aligned}$$

There will be 58.5% left after 600 million years, and the halflife is 780 million years.

14. If you wish, you may use billions of years as your unit. I've chosen to use years, myself.

find r:

$$
\begin{aligned}
P &= P_o e^{-rt} \\
\frac{1}{2}P_o &= P_o e^{-r(7.0\times10^8)} \\
\frac{1}{2} &= e^{-7.0\times10^8 r} \\
\ln\left(\frac{1}{2}\right) &= -7.0\times10^8 r \\
r &= \frac{\ln(1/2)}{-7.0\times10^8} \\
r &= 9.9021\times10^{-10}
\end{aligned}
$$

find A:

$$
\begin{aligned}
P &= P_o e^{-rt} \\
P &= P_o e^{-9.9021\times10^{-10}(1\times10^9)} \\
P &= 0.371499 P_o
\end{aligned}
$$

37% of the original uranium will remain, so 63% must have decayed away.

15. find r:

$$P = P_o e^{-rt}$$
$$\frac{1}{2}P_o = P_o e^{-r(36)}$$
$$\frac{1}{2} = e^{-36r}$$
$$\ln\left(\frac{1}{2}\right) = -36r$$
$$r = \frac{\ln(1/2)}{-36} = 0.019254$$

find P:

$$P = P_o e^{-rt}$$
$$P = P_o e^{-0.019265(2.5\times 60)}$$
$$P = 0.055681P_o$$

Only 5.6% of the original amount of Explodium will be left.

16. find r:

$$P = P_o e^{-rt}$$
$$0.12P_o = P_o e^{-r(11.5)}$$
$$0.12 = e^{-11.5r}$$
$$\ln(0.12) = -11.5r$$
$$r = \frac{\ln(0.12)}{-11.5}$$
$$r = 0.184371$$

find t:

$$P = P_o e^{-rt}$$
$$\frac{1}{2}P_o = P_o e^{-0.184371t}$$
$$\frac{1}{2} = e^{-0.184371t}$$
$$\ln\left(\frac{1}{2}\right) = -0.184371t$$
$$t = \frac{\ln(1/2)}{-0.184371}$$
$$t = 3.75953$$

The halflife is 3.8 days.