

Section 1.10: More Laws of Logic

Answers

1. $p \vee p \Leftrightarrow p$. Rich is seven feet tall.
2. $q \wedge q \Leftrightarrow q$. Susan has brown hair.
3. $\overline{p \vee q} \Leftrightarrow \overline{p \wedge q}$. It is not the case that Rich is seven feet tall and Susan has brown hair.
4. $\overline{p \wedge q} \Leftrightarrow \overline{p} \vee \overline{q}$. Rich is not seven feet tall or Susan does not have brown hair.
5. $\overline{p \vee q} \Leftrightarrow \overline{p} \wedge \overline{q}$. Rich is not seven feet tall and Susan does not have brown hair.
6. $\overline{p \wedge q} \Leftrightarrow \overline{p \vee q}$. It is not the case that Rich is seven feet tall or Susan has brown hair.
7. $p \wedge q \Leftrightarrow q \wedge p$. Susan has brown hair and Rich is seven feet tall.
8. $q \vee p \Leftrightarrow p \vee q$. Rich is seven feet tall or Susan has brown hair.
9. De Morgan's
10. absorption
11. distributive
12. De Morgan's
13. distributive
14. absorption
15. $\overline{A} + \overline{B}$, absorption
16. \overline{AB} , idempotent
17. $B \cap (A \cup C)$, distributive
18. q , absorption
19. 1, complement
20. $A \cup B$, De Morgan's
21. $p \vee (p \wedge q) \Leftrightarrow p$. The moon is made of green cheese.
22. $q \wedge (q \vee p) \Leftrightarrow q$. The earth is made of green cheese.
23. $(q \wedge \overline{p}) \vee p \Leftrightarrow p \vee (\overline{p} \wedge q)$ (note: I'm using the commutative laws to rearrange things)
 $\Leftrightarrow p \vee q$. The moon or the earth is made of green cheese.
24. $q \wedge (p \vee \overline{q}) \Leftrightarrow q \wedge p$. The earth and the moon are made of green cheese.

25.

A	B	$A \oplus B$	\overline{A}	\overline{B}	$\overline{A}B$	$A\overline{B}$	$\overline{A}B + A\overline{B}$
0	0	0	1	1	0	0	0
0	1	1	1	0	1	0	1
1	0	1	0	1	0	1	1
1	1	0	0	0	0	0	0

26. By DeMorgan's law, $\overline{AB} = \overline{A} + \overline{B}$

A	B	$A \text{ NAND } B = \overline{AB}$	\overline{A}	\overline{B}	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

27. 1

28. 1

29. $A + B$

30. AB

31. 1

32. $A\overline{B}$

33. $\overline{A}B$

34. $A + \overline{B}$

35. $A + \overline{B}$

36. 0

37. $A\overline{B} + \overline{A}B$

38. $A + B + C$

39. B

40. $A + B$

Section 1.10 - Proofs

(41)

$$B\bar{B} + AA = A$$

$$0 + AA = A$$

$$AA = A$$

$$A = A$$

complement

identity

idempotent

(42)

$$\bar{A}(B + \bar{B}) = \bar{A}$$

$$\bar{A}(1) = \bar{A}$$

$$\bar{A} = \bar{A}$$

complement

identity

(43)

$$ABC + AB\bar{C} = AB$$

$$AB(C + \bar{C}) = AB$$

$$AB(1) = AB$$

$$AB = AB$$

distributive

complement

identity

(44)

$$AB + \bar{A}B C = AB + C$$

$$(AB) + (\bar{A}B)C = AB + C$$

$$AB + C = AB + C$$

associative

(can skip this step)

absorption

(45)

$$A + AB + ABC = A$$

$$A + ABC = A$$

$$A + A(BC) = A$$

$$A = A$$

absorption

associative

absorption

(can skip)

(46)

$$\bar{A}C + A\bar{B}C = \bar{A}C + \bar{B}C$$

$$(\bar{A} + A\bar{B})C = \bar{A}C + \bar{B}C$$

$$(\bar{A} + \bar{B})C = \bar{A}C + \bar{B}C$$

$$\bar{A}C + \bar{B}C = \bar{A}C + \bar{B}C$$

distributive

absorption

distributive

$$(47) \quad \overline{AB} (A+B) = \overline{A}B + A\overline{B}$$

$$(\overline{A} + \overline{B})(A+B) = \overline{A}B + A\overline{B}$$

de Morgan's

$$\overline{A}A + \overline{A}B + \overline{B}A + \overline{B}B = \overline{A}B + A\overline{B}$$

distributive

$$0 + \overline{A}B + \overline{B}A + 0 = \overline{A}B + A\overline{B}$$

complement

$$\overline{A}B + \overline{B}A = \overline{A}B + A\overline{B}$$

identity

$$(48) \quad \overline{\overline{A}BC} + 0 = \overline{A}BC\overline{0}$$

$$\overline{\overline{A}BC} \overline{0} = \overline{A}BC\overline{0}$$

de Morgan's

$$\overline{A}BC\overline{0} = \overline{A}BC\overline{0}$$

complement

$$(49) \quad A\overline{B}\overline{\overline{A}C} = A\overline{B}$$

$$A\overline{B}(A+C) = A\overline{B}$$

$$A\overline{B}A + A\overline{B}C = A\overline{B}$$

$$A\overline{B} + A\overline{B}C = A\overline{B}$$

$$A\overline{B} = A\overline{B}$$

De Morgan's

distributive

idempotent

absorption