Section 1.10: More Laws of Logic

Exercises

(Note that these are the same exercises as at the beginning of section 1.5, but with a little twist.) Let p be "Rich is seven feet tall" and q be "Susan has brown hair." Translate the following English sentences into logical notation. Then, use one of the laws of logic to write an equivalent logical expression. Finally, translate your new expression back into an English sentence.

1. Rich is seven feet tall or he is seven feet tall.

2. Susan has brown hair and she has brown hair.

3. Either Rich is not seven feet tall or Susan does not have brown hair.

4. It is not true that Rich is seven feet tall and Susan has brown hair.

5. It is not true that Rich is seven feet tall or Susan has brown hair.

6. Rich is not seven feet tall and Susan does not have brown hair.

7. Rich is seven feet tall and Susan has brown hair.

8. Susan has brown hair or Rich is seven feet tall.

Name the law of logic used in the following. Note that the variables have changed, but that the law is still valid.

9. $\overline{q \vee r} \Leftrightarrow \overline{q} \wedge \overline{r}$

10. $\overline{B} \cap (B \cup \overline{A}) = \overline{B} \cap \overline{A}$

11. $(p \wedge q) \vee (p \wedge \overline{q}) \Leftrightarrow p \wedge (q \vee \overline{q})$

12. $\overline{\overline{A} + C} = A\overline{C}$

13. $B + A\overline{C} = (B+A)(B+\overline{C})$

14. $\overline{p} \vee (p \wedge r) \Leftrightarrow \overline{p} \vee r$

Simplify the given expression, and state the name of the law you used. You should be able to do these in a single step.

15. $\overline{A} + A\overline{B}$

16. $\overline{AB} + \overline{AB}$

17. $(A \cap B) \cup (B \cap C)$

18. $q \vee (q \wedge r) \Leftrightarrow q$

19. $\overline{C} + C$

20. $\overline{\overline{A} \cap \overline{B}}$

(Note that these are the same exercises as at the beginning of section 1.5, but with a little twist.) Let p be "The moon is made of green cheese" and q be "The earth is made of green cheese." Translate the following English sentences into logical notation. Then, use one of the laws of logic to write an equivalent logical expression. Finally, translate your new expression back into an English sentence.

21. Either the moon is made of green cheese or both the moon and the earth are made of green cheese.

22. The earth is made of green cheese and either the earth or the moon is made of green cheese.

23. Either the earth is made of green cheese while the moon is not, or the moon is made of green cheese.

24. The earth is made of green cheese and either the moon is made of green cheese or the earth is not.

25. Remembering that $\oplus$ is "exclusive or", show that $A \oplus B = \overline{A}\,B + A\,\overline{B}$ by using a truth table.

26. The NAND gate (not-AND) has the following truth table. Use DeMorgan's laws to find an equivalent Boolean expression using only OR and NOT, and show that your expression has the same truth table.

A	B	A NAND B = $\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

Simplify the following Boolean expressions using the laws of logic. If you're stuck, try using a truth table.

27. $A + \overline{C} + B + \overline{A} + \overline{B}$

28. $A + \overline{B} + A + B + A$

29. $\overline{\overline{A}\ \overline{B}}$

30. $\overline{\overline{A}+\overline{B}}$

31. $\overline{A} + B + A\overline{B}$

32. $A\overline{B}\ \overline{C} + A\overline{B}C$

33. $\overline{A}BC + \overline{A}B\overline{C} + \overline{A}B\overline{D} + \overline{A}BD$

34. $AB + A + \overline{A}\ \overline{B}$

35. $A + \overline{B}CD + \overline{B}$

36. $\overline{A}\ \overline{B}(A + B)$

37. $(\overline{A} + \overline{B})(A + B)$

38. $A + \overline{A}B + \overline{B}C$

39. $B(A + C) + \overline{A}B\overline{C}$

40. $(A + B + C)(A + B + \overline{C})$

Prove that the following Boolean expressions are equivalent by using the laws of logic. If you're stuck, try using a truth table.

41. $B\overline{B} + AA = A$

42. $\overline{A}(B + \overline{B}) = \overline{A}$

43. $ABC + AB\overline{C} = AB$

44. $AB + \overline{AB}C = AB + C$

45. $A + AB + ABC = A$

46. $\overline{A}C + A\overline{B}C = \overline{A}C + \overline{B}C$

47. $\overline{AB}(A + B) = \overline{A}B + A\overline{B}$

48. $\overline{\overline{\overline{A}BC}+D} = \overline{A}BC\overline{D}$

49. $A\overline{B}\overline{\overline{A}\ \overline{C}} = A\overline{B}$