Section 1.9: Laws of Logic

Exercises

1. Which of the following statements is always true?
   a) Darth Vader is both evil and not evil.
   b) Darth Vader is both evil and evil.
   c) Darth Vader is either evil or evil.
   d) Darth Vader is either evil or not evil.

2. Which of the following statements is always false?
   a) The roadrunner has escaped from the wily coyote and he has not escaped from the wily coyote.
   b) The roadrunner has escaped from the wily coyote and he has escaped from the wily coyote.
   c) The roadrunner has escaped from the wily coyote or he has not escaped from the wily coyote.
   d) The roadrunner has escaped from the wily coyote or he has escaped from the wily coyote.

3. Use a truth table to prove that the two idempotent laws are true.

4. Use a truth table to prove that the four identity laws are true.

Name the law of logic used in the following. Note that the variables have changed, but that the law is still valid.

5. $\overline{q} \vee 1 \Leftrightarrow 1$

6. $\overline{\overline{A}} = A$

7. $\overline{r} \wedge r \Leftrightarrow 0$

8. $\overline{p} \vee 0 \Leftrightarrow 0 \vee \overline{p}$

9. $\overline{B} \cap \varnothing = \varnothing$

10. $q \vee q \Leftrightarrow q$

11. $AB + \overline{AB} = 1$

12. $\left(\overline{p} \wedge q\right) \wedge \overline{q} \Leftrightarrow \overline{p} \wedge \left(q \wedge \overline{q}\right)$

Simplify the given expression, and state the name of the law you used. You should be able to do these in one step.

13. $0 \vee r$

14. $C+\overline{C}$

15. $\overline{\overline{p}}$

16. $\overline{A} \cap \overline{A}$

17. $U \cap \overline{B}$

Use the laws of logic to simplify the following logical expressions. If you’re completely stuck, try using a truth table instead.

18. $(p \wedge p) \vee (q \wedge \overline{q})$

19. $(p \vee p) \wedge (q \vee 0)$

20. $p \vee (q \wedge \overline{q})$

Use the laws of logic to simplify the following Boolean expressions. If you’re completely stuck, try using a truth table instead.

21. $(A + A)(B + \overline{B})$

22. $B \cdot 0 + AA$

23. $(B + \overline{B})(A + 1)$

24. $AB\overline{B}$

Prove the following Boolean expressions are equivalent using the laws of logic. If you’re stuck, try using a truth table.

25. $(A\overline{A})\overline{B} = A(B\overline{B})$

26. $B \cdot 1 + A\overline{A} = \overline{\overline{B} \bullet 1}$

27. $(A + 0)(B + \overline{B}) = A$

28. $AA + \overline{B}\,\overline{B} = A + \overline{B}$