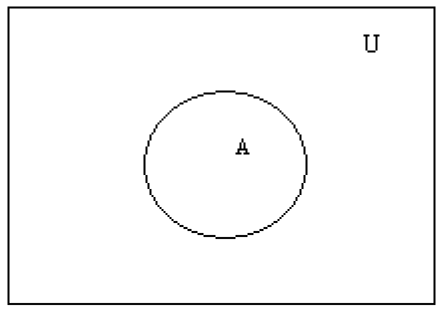
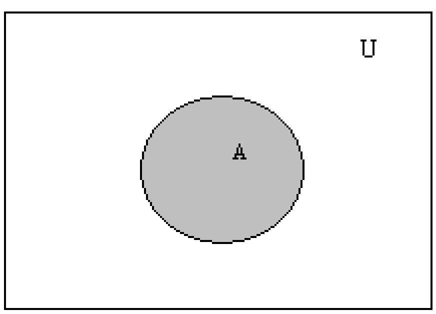
Section 1.4: Venn Diagrams

Venn Diagrams with One Set A

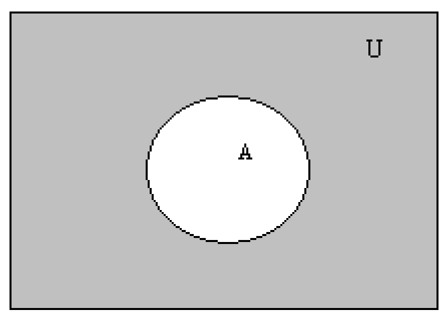
As discussed in the previous section that if we wish to consider the set $\overline{A}$, we need to know A and the universal set U that A is drawn from. We can represent the set A and operations on it using a Venn diagram as shown below.



In this diagram, the rectangle stands for the universal set U, while the circle denotes the set A. Anything located inside the circle is a member of A, while anything outside is not in A but is in U. (We don't consider what's going on outside U, since our universe has shrunk down to the contents of the universal set U!) So, we can colour in the diagrams to show A and $\overline{A}$ as shown in the following.



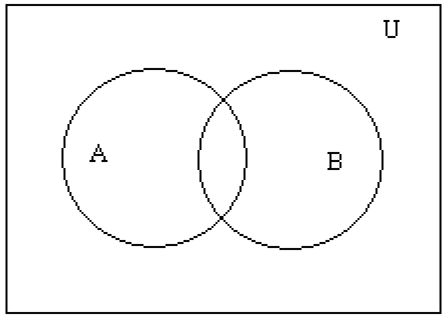
A



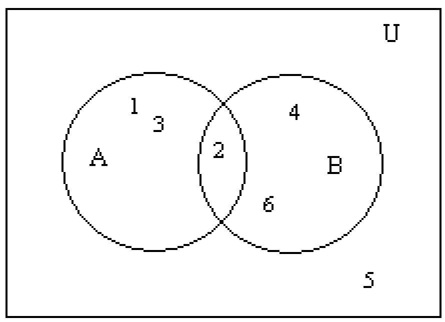
$\overline{A}$

Venn Diagrams with Two Sets, A and B

Venn diagrams with only one set don't generally contain much information, as it's usually pretty easy to visualize what A and $\overline{A}$ mean when you only have the one set. Where it gets more interesting is when you have sets A and B in the same diagram, as you can see in the next diagram.

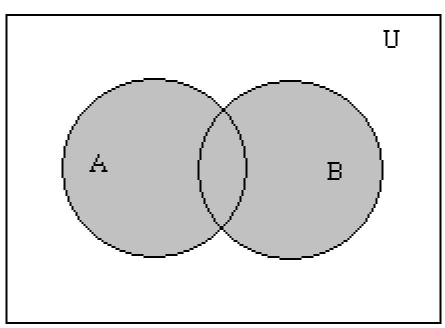


The Venn diagram is a very general diagram that works for all sets. So that you are able to better visualize how this works for sets composed of natural numbers, consider the sets A = {1, 2, 3} and B = {2, 4, 6} with universal set U = {1, 2, 3, 4, 5, 6}. Then if we were to place these elements into their appropriate places on the Venn diagram, it would look like this.



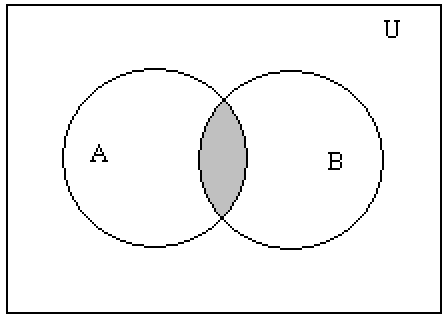
Notice that since the element 1 appears only in A, it is inside the circle for A but outside the circle for B. Since 2 is common to both A and B, it is inside both circles. The number 5 is in neither A nor B, so it is placed inside the rectangle for U but outside the circles for A and B.

Let's try doing some shading to represent operations on the sets A and B. For example, to show $A \cup B$, you would shade in all of the A circle and all of the B circle, since the $\cup$ operator gives you elements in either A **or** B.



$A \cup B$

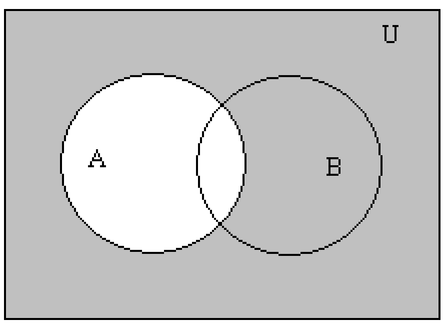
To show A ∩ B, you need to show those regions common to both. This translates to shading the little slice of overlap between the two circles.



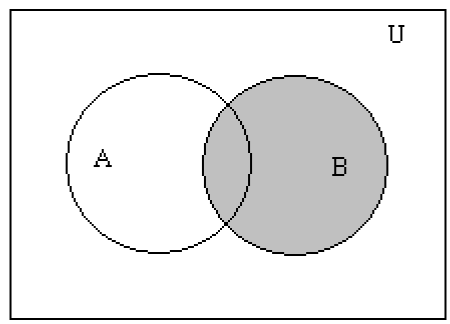
$A \cap B$

More complications

Suppose you wished to shade in a Venn diagram to show $\overline{A} \cap B$. The best approach might be to do it by steps:

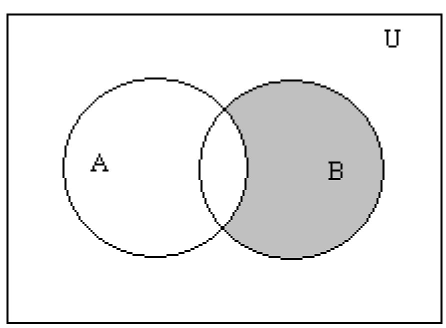


$\overline{A}$



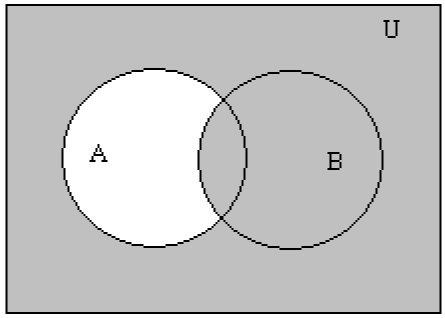
B

And then the intersection of these two diagrams requires (since it's an intersection) shading all the regions that are shaded in **both** of the above diagrams.



$\overline{A} \cap B$

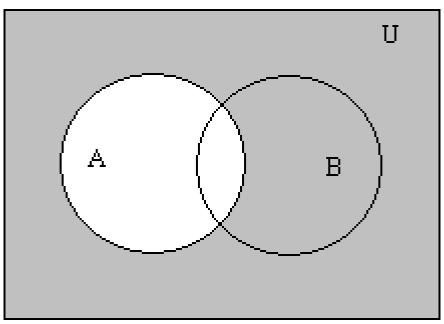
To find $\overline{A} \cup B$, you'd take the diagrams for $\overline{A}$ and B, and then shade in the regions that are shaded in for **either** of the above diagrams.



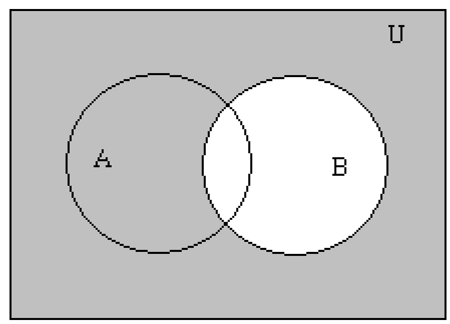
$\overline{A} \cup B$

De Morgan's Laws

Let's find out what the diagram looks like for $\overline{A} \cup \overline{B}$. Here's $\overline{A}$ and $\overline{B}$ below.

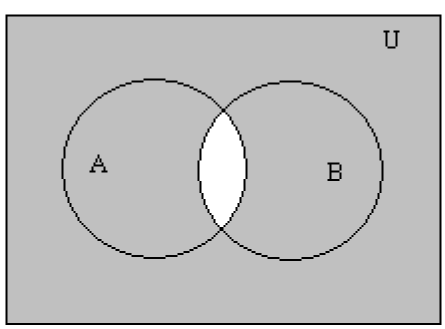


$\overline{A}$



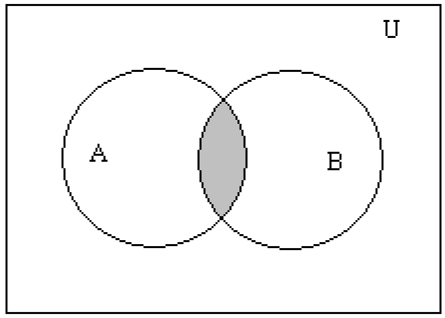
$\overline{B}$

We need to shade in regions that are shaded in for **either** of the above diagrams, to get the following.



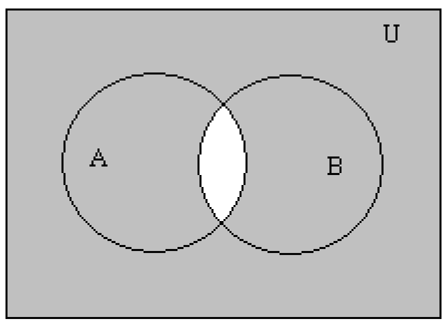
$\overline{A} \cup \overline{B}$

Notice, however, that we get the same thing if we consider $\overline{A \cap B}$. Here's the diagram for A∩B:



A∩B

To get $\overline{A \cap B}$ from A∩B, we take the A∩B and negate it. Essentially, we "reverse" the diagram by shading in all previously unshaded regions, and not shading in any previous shaded regions, resulting in the following.



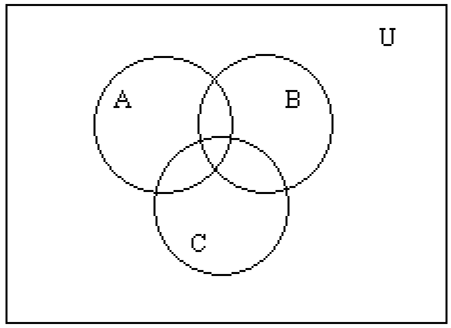
$\overline{A \cap B}$

You can see that we get exactly the same result as when we found $\overline{A} \cup \overline{B}$.

This result, that $\overline{A} \cup \overline{B} = \overline{A \cap B}$, is true for all sets A and B. You could, if you wish, show also that $\overline{A} \cap \overline{B} = \overline{A \cup B}$ for all A and B. These two properties of sets are called De Morgan's theorems and we will be revisiting them later.

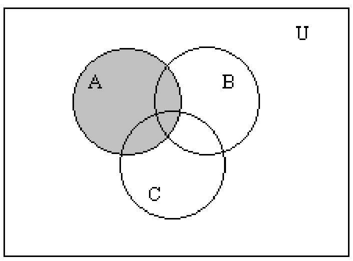
Venn Diagrams with Three Sets: A, B, and C

Similarly, we can do Venn diagrams with three sets, as shown in the next diagram.

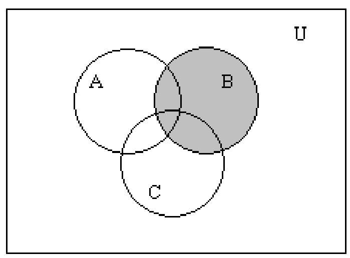


Notice that there is a circle for each set, and that there are regions where some or all of the sets overlap.

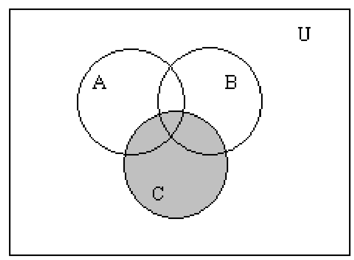
To find out how to shade the diagram for combinations of sets such as (A∪B)∩C, do the shading process in steps. Here's A, B, and C below.



A

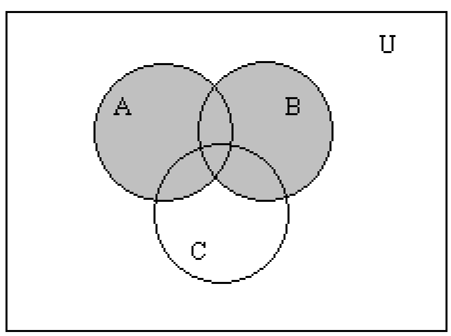


B



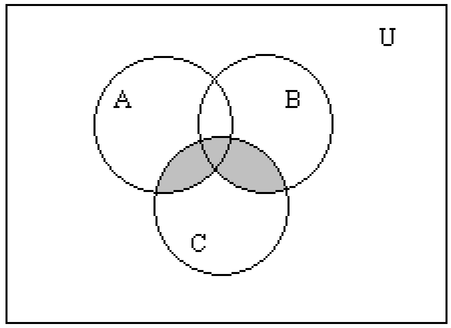
C

Then (A∪B) gives



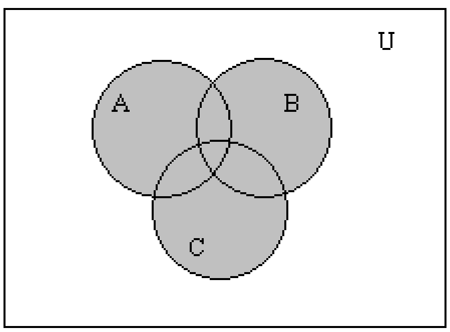
A∪B

and intersecting this with set C from above gives

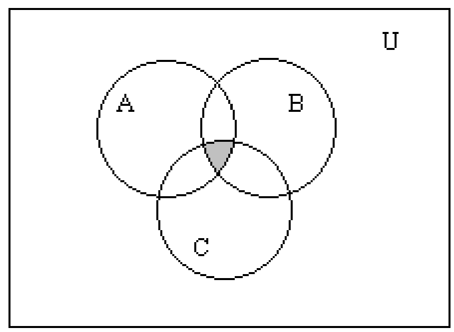


(A∪B)∩C

The diagrams for A∪B∪C and A∩B∩C are then given below. (Because the operations are all the same in each expression, I don't need brackets to show the order of operations for these particular cases.)



A∪B∪C



A∩B∩C