Section 2.1: Relations

Ordered pairs and n-tuples

We have seen that a set is a collection of elements in no particular order. However, if we wish to describe relationships between elements, we will often write them as ordered pairs, (x,y), such as

$$(3, -2) \text{ or}$$

(oranges, $1.29/lb)

Just as in graphing, the ordered pair (3, -2) is not the same as (-2, 3).

An ordered triple then has three elements, (x,y,z), while the more general **n-tuple** is an ordered collection of many elements, such as

$$(a, b, c, d) \text{ or}$$

(computer, Dell, 3.0 GHz, 512 MB RAM, Windows XP).

Relations

We can now define the terms relation and function.

relation = a set of ordered pairs or (more generally) **n-tuples**

Relations can then be defined by listing all of the pairs or n-tuples using set notation. Another way is to list the relation in a table.

Example

Are the following two relations equal?

{(truck, blue), (truck, red), (car, red), (SUV, green)}

vehicle	**colour**
truck	blue
truck	red
car	red
SUV	green

Answer: Yes, because both relations have the same set of ordered pairs.

Relations can also be defined by listing the two sets and the relationship between them.

Example

Let $x \in \{2, 4, 6\}$ and $y \in \{1, 2, 3\}$. $(x,y) \in$ A if $x + y \leq 5$. Find A.

(Here, the definition of A looks a little tricky. It is read aloud as "an ordered pair (x,y) is a member of A if $x + y \leq 5$." So you are being asked to find all sets of ordered pairs that satisfy the equation.)

Answer: A = {(2,1), (2,2), (2,3), (4,1)} or

x	y
2	1
2	2
2	3
4	1

Functions

In general, functions are defined using n-tuples. However, for this course we will only consider functions of ordered pairs.

function = a relation which has for each value of the first component, exactly one value of the second component

Example

Is the relation {(truck, blue), (truck, red), (car, red), (SUV, green)} also a function?

Answer: No. When the first component is a truck, there are two possible second components, blue and red.

Example

Let $x \in \{2, 4, 6\}$ and $y \in \{1, 2, 3\}$. $(x,y) \in$ A if $x/y = 2$. Is A a function?

Yes. A = {(2,1), (4,2), (6,3)}. For each x value, there is only one possible y value, so A is a function.

Example

Let $x, y \in$ real numbers. $(x, y) \in$ A if $y = 2x - 3$. Is A a function?

Answer: Yes. For every x, plugging it into $y = 2x - 3$ gives a unique value of y. If you prefer, you can graph the equation (it's a straight line) and use the "vertical line" test to see that it's a function.

Note: to use the "vertical line test", draw a graph of the relation. If a vertical line cuts the graph at only one point anywhere along the graph, the relation is a function.

Cartesian Product

If X and Y are both sets, then the Cartesian product X × Y is the set of all ordered pairs (x,y), where x is an element of set X and y is an element of set Y.

Example

If A = {a, b} and B = {0, 1}, then find A × B and B × A.

Answer: A × B = { (a, 0), (a, 1), (b, 0), (b, 1) }, and

B × A = {(0, a), (0, b), (1, a), (1, b)}.

Because order matters, notice that A × B ≠ B × A unless A = B.

Example

If A = {a, b} and B = {0, 1}, then find A × A and B × A × A.

Answer: A × A = { (a, a), (a, b), (b, a), (b, b) }, and

B×A×A = {(0,a,a), (0,a,b), (0,b,a), (0,b,b), (1,a,a), (1,a,b), (1,b,a), (1,b,b)}.

Functions and relations between two sets are then **subsets** of the Cartesian product.

Example

Let A = {1, 2} and B = {1, 2, 3}. Is A × A ⊆ B × A? Is A × A a function?

Answer:

A × A = {(1,1), (1,2), (2,1), (2,2)}

B × A = {(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)}

Yes, $A \times A \subseteq B \times A$, because every member of $A \times A$ is also in $B \times A$, and there is at least one member of $B \times A$ that isn't in $A \times A$. For example, (3,1) isn't in $A \times A$.

No, $A \times A$ is not a function because for $x = 1$, there are two possible y's, 1 and 2.

Example

Let $X = \{1,2\}$ and $Y = \{1,3\}$. $(x,y) \in R$ if $x + y \geq 4$. Is R a function? Is $X \times X \subseteq X \times Y$?

Answer: $R = \{(1,3), (2,3)\}$. Yes, R is a function, because for each x there is only one y.

No, $X \times X$ is not a subset of $X \times Y$ because, for example, (1,2) is in $X \times X$ but not in $X \times Y$.