Section 3.3: Geometric Sequences and Series

Solutions

1. no
2. no
3. yes, $r = 2$
4. no
5. no
6. yes, $r = \frac{2}{3}$
7. $a_n = (3)^{n-1}$ and $\begin{cases} a_1 = 1 \\ a_n = 3a_{n-1} \end{cases}$
8. $a_n = 64\left(\frac{1}{4}\right)^{n-1}$ and $\begin{cases} a_1 = 64 \\ a_n = \frac{a_{n-1}}{4} \end{cases}$
9. $a_n = 2(-3)^{n-1}$ and $\begin{cases} a_1 = 2 \\ a_n = -3a_{n-1} \end{cases}$
10. $a_n = 24(0.1)^{n-1}$ and $\begin{cases} a_1 = 24 \\ a_n = 0.1 \times a_{n-1} \end{cases}$
11. $a_n = 12\left(\frac{3}{2}\right)^{n-1}$, so $a_{50} \approx 5.1 \times 10^{9}$ and $a_{261} \approx 7.3 \times 10^{46}$
12. $a_n = 12\left(\frac{2}{3}\right)^{n-1}$, so $a_{50} \approx 2.8 \times 10^{-8}$ and $a_{261} \approx 1.97 \times 10^{-45}$
13. no
14. yes, with $r = 2$
15. yes, with $r = 10$
16. no

17. yes, with $r = -1$

18. no

19. $a_n = 5(3)^{n-1}$, so $a_{201} = 5(3)^{200} = 1.33 \times 10^{96}$

20. $a_n = 7(-2)^{n-1}$, so $a_{20} = 7(-2)^{19} = -3{,}670{,}016$

21. S_{20} = 200 (The exact answer is $\frac{26214375}{131072}$ or 1.99980926513671875, but if you round to three decimals, the answer is 200.000.)

22. $S_{20} = 104{,}857{,}500$

23. $S_\infty = \frac{a_1}{1-r} = \frac{-6}{1-(-2/3)} = -\frac{18}{5} = -3.6$

24. $S_\infty = 200$

25. S_∞ does not exist ($r > 1$)

26. $S_\infty = 16$

27.
$$\begin{aligned} S_{11} &= 2^2 + 2^3 + 2^4 + \ldots + 2^{12} \\ &= \frac{a_1(1-r^n)}{1-r} = \frac{2^2(1-2^{11})}{1-2} \\ &= 8188 \end{aligned}$$

28. $S_\infty = 22.5$

29. $S_\infty = \frac{5}{18} = 0.2\overline{7}$

30. S_∞ does not exist ($r < -1$)

31. 3 years is 36 months, so we have a 36-term sequence starting with 1, 2, 4, 8, … The nth term will be $a_n = 1(2)^{n-1}$, so the 36th term will be $a_{36} = 1(2)^{35} = 34{,}359{,}738{,}368$, which is a tad larger than the total population of Transylvania.

32. 1 man

7 wives

sacks = #wives × # sacks/wife = 7 × 7 = 49

cats = #sacks × # cats/sack = 49 × 7 = 343

kits = #cats × # kits/cat = 343 × 7 = 2401

So kits, cats, sacks, and wives is 2401, 373, 49, 7, which is a geometric sequence with four terms: $a_1 = 2401$ and $r = 1/7$.

33. After one fold, the thickness will be 0.097×2, after two folds 0.097×2^2, etc. So our starting term will be 0.097×2 and then will double with $r = 2$ thereafter.

So $a_n = 0.097(2)^n$, and the 7[th] term will be $a_7 = 0.097(2)^7 = 12.416$, and the paper thickness will be 12.4 mm, or just over 1 cm thick.

(The Mythbusters realized that the problems with paperfolding lie with the fold itself. If I remember correctly, they resorted to C-clamps and hitting the fold with a hammer to flatten it.)