Section 6.1: Counting Techniques

Solutions

1. First, note that 2-digit numbers run from 10, 11, 12, … 99.

   a) The even ones are 10, 12, 14, … 98. You can do the really short method to count them: ___ ___ – the first slot can have the digits 1-9 for 9 choices, and the second can only have 2, 4, 6, 8, or 0 for 5 choices. Then the total number is $9 \times 5 =$ 45 numbers.

   b) Unfortunately, you cannot use the above technique for dividing by 7, since 7 doesn't restrict the last digit. Instead, you have to note that the first 2-digit number that's divisible by 7 is 14, the next is 21, then 28, and so on. To find the last digit, you have to count backwards from 99 to find one that's divisible by 7. 99 does not divide evenly by 7, but with your calculator (sigh) you can quickly find that $98 \div 7 = 14$.

      So our sequence is 14, 21, 28, … 98. This is just 2×7, 3×7, 4×7, … 14×7. So there are (last – first + 1) = 14 – 2 + 1 = 13 numbers divisible by 7.

   c) Total number of 2-digit numbers: last – first + 1 = 90 – 10 + 1 = 90. So the total number of 2-digit numbers **not** divisible by 7 is the total number minus the number that **are** divisible by 7. So, we get 90 – 13 = 77 for our answer.

2. First, note that 4-digit numbers run from 1000, 11, 12, … 9999.

   a) The first number that's divisible by 3 is 1002, the next is 1005, then 1008, and so on up to 9999, which also divides evenly by 3.

      So our sequence is 1002, 1005, 1008, … 9999. This is just 334×3, 335×3, 336×3, … 3333×3. So there are (last – first + 1) = 3333 – 334 + 1 = 3000 numbers divisible by 3.

   b) Numbers that divide evenly by 5 end in either 0 or 5. You can do the really short method to count them: ___ ___ ___ ___ – the first slot can have the digits 1-9 for 9 choices, the second and third slots can have 0-9 for 10 choices and the second can only have 0 or 5 for 2 choices. Then the total number is $9 \times 10 \times 10 \times 2 =$ 1800 numbers.

   c) Numbers divisible by 3 **and** 5 must be divisible by 15. Looking at our sequence in a), we can see that 1005 must be the first number, then add 15 to get 1020, etc. Starting from 9999 and working downwards, we'll see that the first possibility is 9995, which doesn't divide, but 9990 does.

So our sequence is 1005, 1020, 1035, … 9990. This is just 67×15, 68×15, 69×15, … 666. So the total number is 666 – 67 + 1 = 600.

d) n(3 or 5) = n(3) + n(5) – n(3 and 5)
= 3000 + 1800 – 600
= 4200

e) total number with 4-digits: 9999-1000+1 = 9000 numbers
Then the total divisible by neither 3 nor 5 is 9000 – 4200 = 4800.

3. Case-sensitive, alpha-numeric passwords have 2×26 + 10 choices for characters, or 62 different possibilities. The number of passwords containing 4 characters is 62×62×62×62 = 14,776,336. The number of passwords containing 5 characters is 62×62×62×62×62 = 916,132,832. The total number of passwords is then the sum of these two (since you can't have four **and** five at the same time), = 930,909,168.

4. a) This is the same as in question #3: 916,132,832.

b) If you can't repeat, then you get 62 choices for the first one, 61 for the second, etc., to give 62×61×60×59×58 = 776,520,240.

c) If the first number must be a letter, then you only have 52 possibilities for the first slot: 52×62×62×62×62 = 768,369,472.

5. a) You have 62 choices for each slot, so result is $62^8 = 2.18\times10^{14}$.

b) The number containing **no** digits is $52^8 = 5.35\times10^{13}$. So the number containing at least one digit is $2.18\times10^{14} – 5.35\times10^{13} = 1.65\times10^{14}$.

c) The number containing **no** letters is 10^8. So the number containing at least one letter is $2.18\times10^{14} – 10^8 = 2.18\times10^{14}$ (essentially the same number, since 10^8 is so much smaller).

d) The number containing at least one digit and one letter must be the total minus (the number containing no digits plus the number containing no letters). So we get $2.18\times10^{14} – 5.35\times10^{13} – 10^8 = 1.65\times10^{14}$ (very close to the answer to b – you'd have to write out a few more decimals to see the difference).

6. a) If no "A"s are allowed, then we are constrained to 61 choices from our original 62. Then we'll get $61^6 = 5.15\times10^{10}$ passwords.

b) This will again give us 61 choices for each character, or $61^6 = 5.15\times10^{10}$ passwords.

c) Now, we're down to 60 choices, since we can't have "A" or "a". We then get $60^6 = 4.67\times10^{10}$ passwords.

7. Peter has assigned 25-37 pages, so 25, 26, 27, … 37. # pages = last – first + 1 = 37 – 25 + 1 = 13 pages.

8. Gilles has assigned odd questions, so 7, 9, 11, … 89. It's a bit tricky to do the odd numbers, so I'm going to take all numbers from 7 to 89 and subtract the even numbers.

   total number from 7 to 89: $89 - 7 + 1 = 83$

   even numbers: 8, 10, 12, … 88 is the same as 4×2, 5×2, 6×2, … 44×2. So we get $44-4+1 = 41$ even numbers

   odd numbers = $83 - 41 = 42$ odd numbered questions

9. first letter: 13 choices, second and third letters: 24 choices, all numbers: 10 choices

   So we get _13_ _10_ _24_ _10_ _24_ _10_ = $13\times10\times24\times10\times24\times10 = 7{,}488{,}000$ possible postal codes. (Note that since postal codes reference a geographical area and not a group of people, we're not likely to run out any time soon!)

10. a) Counting on my fingers, I get that Tuesday, Thursday, and Saturday contain the letter "t" for a total of 3.

    b) Counting on my fingers, I get that all days except for Monday and Friday have "s" in them for a total of 5.

    c) I see that all of the days containing "t" also contain "s" for a total of 3.

    d) Using my counting rules, I get that n(t or s) = n(t) + n(s) – n(t & s) = 3 + 5 – 3 = 5.

11. I will not be using 3 letters, leaving 23 lower-case letters to choose from. But they have to be different, so I'll get 23×22 choices = 506 choices.

12. __26__ __25__ __10__ __10__ __10__ = $26\times25\times10^3 = 650{,}000$.