Section 6.3: Probability

Solutions

1. A fair twelve-sided die is rolled.

   a) P(7) = 1/12 (only one way to get a 7, and there are 12 outcomes)
   b) Even numbers from 1 to 12: 2, 4, 6, 8, 10, 12, so six possibilities out of 12 outcomes. P(even) = 6/12 = ½. (Or you could note that exactly half of the outcomes gave an even number to get an even shorter solution.)
   c) P(>5)=P(6 or 7 or 8 or 9 or 10 or 11 or 12) = 7/12
   d) P(not 7) = 1 – P(7) = 11/12
   e) P(1 or 2) = 2/12 = 1/6

2. I'm going to use the brute force method here and list all possible rolls:

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

   a) We can see that there are sixteen possibilities in total, and four of them will result in the same number on both dice, so P(both same) = 4/16 = ¼.
   b) P(different) = 1 – P(same) = ¾.
   c) P(sum of 6) = 3/16 (need 42, 33, or 24).
   d) You can count them up to find P(at least one 3) = 7/16.
   [Or you could say that's P(at least one 3) = 1 – P(no 3s). And the number of rolls with no 3s is _3__ _3___= 9 possibilities, so then you'd get 1 – 9/16 = 7/16.]

3. P(2 digits divisible by 7) = n(2 digits divisible by 7) / n(2 digits) = 13/90.

4. We found from 4.2 that there are ${}_6C_3 = 20$ different possible sets of 3 cards from six. So if the person claiming to have ESP guesses randomly, he has a 1/20 chance.

5. This is a permutation question, since order matters.

   a) You can either just calculate ${}_3P_3 = 6$ or list the possible outcomes: {ABC,ACB,BAC,BCA,CAB,CBA}. Since it's the same soft drink in each glass, the lists should all be equally probable at 1/6 each.
   b) Only 2 of the 6 outcomes have A first, so P(A first) = 2/6 = 1/3. Or you could say that if A is first, there are two choices for second place and one for third (or ${}_2P_2 = 2$, if you insist).
   c) If either B or C is ranked first, then A is not. So P(B or C) = 1 – P(A) = 2/3.
   d) If A is first and B is last, then C is in the middle. P(ACB) = 1/6.

6. Your ATM/debit card has a four-digit PIN number associated with it. If there are no restrictions on what digits or what order you can pick them, then

   a) There are 10^4 PIN numbers possible (or 10,000).
   b) If that person is only given one guess (I should have said that!), then their chance is 1/10,000.
   c) If they know the first two numbers, then there are only 10×10 possible PIN numbers left. So their chances are now 1/100.

7.

	Gandalf	Dumbledore	total
CST	90	10	100
English	40	60	100
total	130	70	200

   a) P(G) = 130/200 = 13/20 (or 65%)
   b) P(C|G) = n(CG)/n(G) = 90/130 = 9/13 (which is roughly 69%)
   c) P(G|C) = n(CG)/n(C) = 90/100 = 9/10 (or 90%)
   d) P(E or D) = (40 + 60 + 10)/200 = 11/20 (or 55%)

8.

	Windows	Linux	total
faculty	6	2	8
students	24	8	32
total	30	10	40

   a) P(L)=10/40 = ¼ or 25%
   b) P(L|S) = 8/32 = ¼ or 25%
   c) Yes, “student” and “Linux user” are independent because P(L) = P(L|S).

9. One thousand television watchers from BC and Alberta were asked if they watched the Rick Mercer Report on CBC with the following results.

	Yes	No	total
BC	500	500	1000
AB	250	750	1000

total	750	1250	2000

a) P(BC or Y) = (500 + 500 + 250)/2000 = 5/8 (or 62.5%).
b) P(BC and Y) = 500/2000 = ¼ (or 25%).
c) P(AB and N) = 750/2000 = 3/8 (or 37.5%)
d) P(BC|Y) = n(BC and Y)/n(Y) = 500/750 = 2/3
e) P(Y|BC) = n (Y and BC)/n(BC) = 500/1000 = ½ (or 50%)

10.

	coffee	tea	other	total
Starbucks	45	9	6	60
Moka House	30	8	2	40
total	75	17	8	100

a) P(C) = 75/100 = ¾ = 75%. P(C|S) = 45/60 = ¾ = 75%. Yes, these events are independent. (You could alternatively calculate P(S)=60% and P(S|C)=60% to reach the same conclusion.)
b) P(T) = 17/100 = 17%. P(T|M) = 8/40 = 1/5 = 20%. As these are not the same, the events are dependent.

11.

a)

	exercise regularly	don't exercise regularly	total
healthy diet	45	30	75
unhealthy diet	15	10	25
total	60	40	100

b) P(E $\overline{H}$) = 15/100 = 15%.
c) P(E or H) = (15 + 45 + 30)/100 = 90/100 = 90%.
(or P(E or H) = 1 – P($\overline{E}\,\overline{H}$) = 1 – 10/100 = 90%)

d) P(H) = 75/100 = 75%. P(H|E) = 45/60 = 75 %. Since these probabilities are the same, eating a healthy diet is independent of exercising regularly for this sample of Canadians.