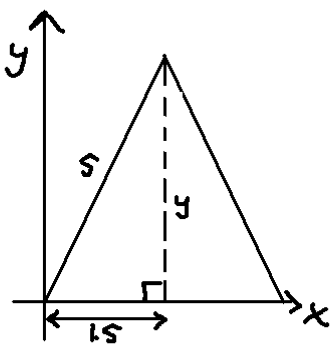
Section 4.3: Applications to Computer Graphics

Solutions

1. The triangle is isosceles, since the two sloping sides are the same length. Dropping a dotted line straight down to the base from the vertex will therefore cut the base in half, making a right triangle as shown below.

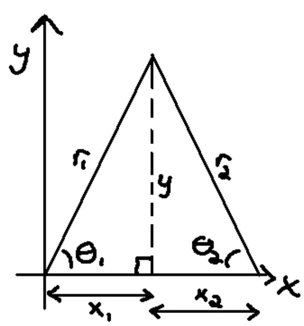


We can then find y from the Pythagorean theorem:

$$a^2 + b^2 = c^2$$
$$(1.5)^2 + y^2 = 5^2$$
$$y^2 = 5^2 - 1.5^2$$
$$y = 4.77$$

The coordinates of the vertex are then (1.5, 4.8).

2. Unlike the previous problem, this triangle is not isosceles. Then $x_1 = 1.8$ and $x_2 = 1.2$ and $y = 3.5$, from the information given in the problem.



Using the Pythagorean theorem to get the two hypotenuses:

$$c^2 = a^2 + b^2 \qquad\qquad c^2 = a^2 + b^2$$
$$r_1^2 = x_1^2 + y^2 \quad \text{and} \quad r_2^2 = x_2^2 + y^2$$
$$r_1^2 = (1.8)^2 + (3.5)^2 \qquad r_2^2 = (1.2)^2 + (3.5)^2$$
$$r_1 = 3.94 \qquad\qquad r_2 = 3.7$$

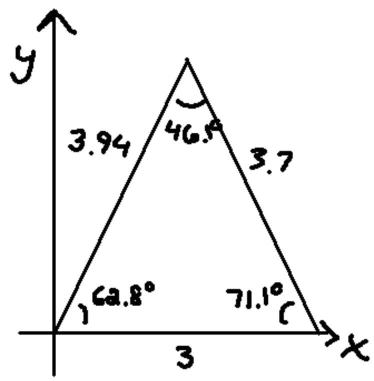
We can find the angles θ_1 and θ_2 using right-triangle trig:

$$\tan\theta_1 = \frac{y}{x_1} = \frac{3.5}{1.8} \quad \text{and} \quad \tan\theta_2 = \frac{y}{x_1} = \frac{3.5}{1.2}$$

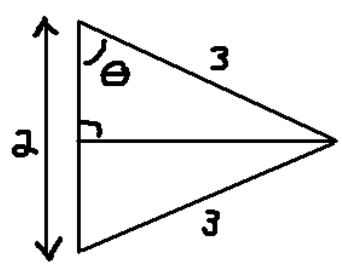
$$\theta_1 = 62.8^\circ \qquad \theta_2 = 71.1^\circ$$

The third angle at the top may be found by subtracting the sum of the other two from 180°: $\theta_3 = 180^\circ - \theta_1 - \theta_2 = 180^\circ - 62.8^\circ - 71.1^\circ = 46.1^\circ$.

Our triangle, therefore, looks like this:



3. Blowing up the triangle forming the point of the crayon, we get the following diagram.



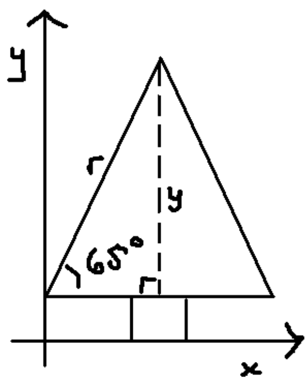
The horizontal line splits the vertical side in two, so the side adjacent to θ equals 1. Then the opposite side may be found by the Pythagorean theorem to be $2\sqrt{2}$, approximately 2.83. To calculate θ, use right-triangle trig:

$$\cos\theta = \frac{adj}{hyp} = \frac{1}{3}$$

$$\theta = 70.5^\circ$$

So the angle θ in the diagram is 70.5° and the coordinates of the point of the triangle will be (4 + 2.83, 1) = (6.83, 1).

4. Dropping a dotted line down from the top of the triangle, we get a right triangle as shown below. The length of the triangle adjacent to the 65° angle is 3, since the base of the triangle is 6 units long.



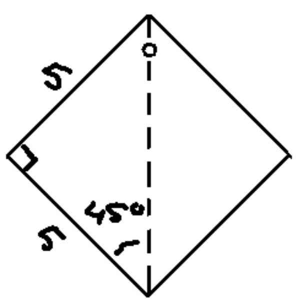
The side y may be found using right-triangle trig:

$$\tan 65^\circ = \frac{y}{3}$$

$$y = 3\tan 65^\circ \approx 6.43$$

The coordinates of the triangle's vertices are therefore (0,2), (6,2), and (3, 8.43).

5. We can make a right triangle with 45° angles by dropping a vertical line from the top of the pendent to the bottom. The resulting isosceles right triangle is shown in the diagram.



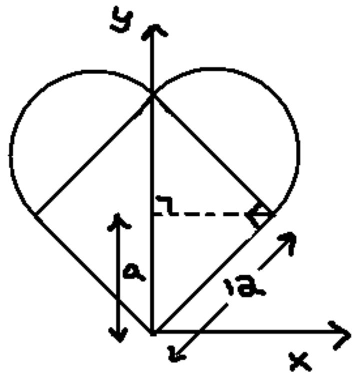
We can then find the hypotenuse by the Pythagorean theorem:

$$c^2 = a^2 + b^2 = 25 + 25$$

$$c = \sqrt{50} = 5\sqrt{2} \approx 7.07$$

The centre of the circle is therefore either $5\sqrt{2} - 1$ units or approximately 6.07 units from the bottom of the pendant.

6. Making a right triangle using one of the sides of the original square as the hypotenuse, we find that the resulting triangle is isosceles (two sides equal).

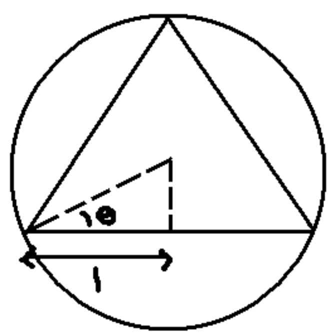


Letting this side equal *a*, then we can find its length using the Pythagorean theorem:

$$a^2 + b^2 = c^2$$
$$2a^2 = 144$$
$$a^2 = 72$$
$$a = \sqrt{72} = 6\sqrt{2} \approx 8.49$$

The coordinates of the centre of the right-hand semicircle can then be found in terms of *a*. The centre will be located halfway along the hypotenuse of the right triangle, so will have an *x*-coordinate equal to half of *a*, while the *y*-coordinate will be one-and-a-half times *a*. Since $a = 6\sqrt{2}$, then the centre of the right semicircle will be at $\left(3\sqrt{2}, 9\sqrt{2}\right)$, which is approximately equal to (4.24, 12.73). The left semicircle will have the opposite *x*-coordinate and the same *y*-coordinate: (–4.24, 12.73).

7. Let's make a right triangle by dropping a line from the centre of the circle/triangle to the bottom side and to the left corner of the triangle, as shown below.



The angle θ is half of the 60° angle making up one of the angles of this equilateral triangle. The hypotenuse of this triangle will be equal in length to the radius of the circle. Using the Pythagorean theorem,

$$\cos 30^\circ = \frac{1}{r}$$
$$r = \frac{1}{\cos 30^\circ} = 1.15$$

so the radius of the triangle is 1.15 units long ($\frac{2\sqrt{3}}{3}$ exactly).