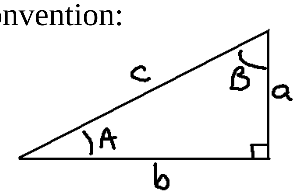
Section 4.2: Applications of Right Triangles

Solving Triangles

In order to complete solve a triangle, you need three pieces of information, one of which must be a side. If we apply this to a right triangle, recall that we must then already know one angle (the right angle). We then need only two further pieces of information to solve the triangle, but as before, one of those pieces must be a side.

To **completely solve a right triangle**, then, means to take those pieces of information and calculate all remaining sides and angles. Before we do that, however, we need to look at one more naming convention:



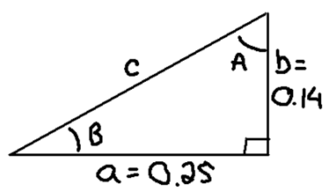
As before, the hypotenuse is c and the two other sides are a and b. The two acute angles are called A and B, and angle A is located opposite side a and so on. Side c, being the hypotenuse, must always be opposite the right angle.

Example

Solve the right triangle with $a = 0.25$ and $b = 0.14$. Round any approximate answers to two decimal places.

Answer

The diagram is shown below.



To get c, we use the Pythagorean theorem:

$$c^2 = a^2 + b^2$$

$$c^2 = (0.25)^2 + (0.14)^2$$

$$c = 0.286531 = 0.29$$

Then we use a ratio of sides to determine one of the two unknown angles. At this point, because all sides are known, any of the three basic trig functions may be used. I will choose to use the tangent, since that will require only the original information given in the problem. Please note that if you use your value of *c* in further calculations, it's good practice to take a few extra decimal places (not just the 0.29 value) in order to minimize round-off errors.

$$\tan B = \frac{b}{a}$$
$$B = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{0.14}{0.25}\right) = 29.2488^\circ = 29.25^\circ$$

Then we can just subtract *B* from 90° in order to determine *A*:

$$A = 90^\circ - B = 90^\circ - 29.25^\circ = 60.75^\circ$$

So the solution to the triangle is

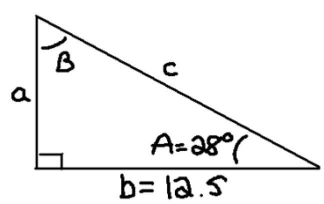
$$A = 60.75^\circ, B = 29.25^\circ, c = 0.29$$

Example

Solve the right triangle with *b* =12.5 and *A* = 28°. Round any approximate answers to one decimal place.

Answer:

The diagram is sketched below.



Firstly, we can find *B* by subtracting *A* from 90°:

$$B = 90^\circ - A = 90^\circ - 28^\circ = 62^\circ$$

We cannot initially use the Pythagorean theorem to determine another side because we only have one of the three sides to begin with. Therefore, we will have to start with a trig ratio and solve for an unknown side. If I want to calculate *c* from the information given, then

$$\cos A = \frac{b}{c}$$
$$c \cos A = b$$
$$c = \frac{b}{\cos A} = \frac{12.5}{\cos 28^\circ} = 14.1571 = 14.2$$

Similarly,

$$\tan A = \frac{a}{b}$$
$$b \tan A = a$$
$$a = b \tan A = 12.5 \tan 28^\circ = 6.64637 = 6.6$$

So the solution to the triangle is

$$a = 6.6, c = 14.2, B = 62^\circ$$