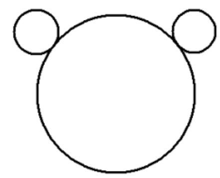
Section 4.3: Applications in Computer Graphics

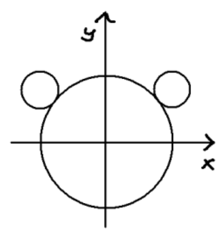
Finding coordinate points

Suppose you were designing a web page and wanted to make a little graphic of, say, Mickey Mouse or Winnie the Pooh, or your favourite cartoon mouse/bear. You might start out with a sketch that looks like the following.

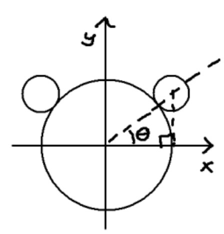


How you'll actually implement this depends on what computer program you are using. Let's work on some tools so that no matter what input is required, you can determine from your diagram any pieces of information that the software might require.

The first thing you might want to do is to impose a coordinate system onto your drawing. You may choose, as we've done here, to put the origin in the middle of your diagram. Another popular choice is to put the origin in the lower left corner. Which you'll use depends again on the software you're using. For the purposes of this course, it doesn't matter which system you pick.

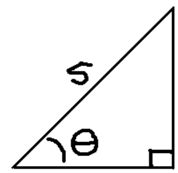


Now, the coordinates of the big circle will be (0,0) because of my choice of origin. However, we'll need to do a bit of work to determine the coordinates of the centres of the two smaller circles.

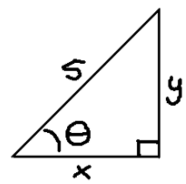


As shown in the diagram above, you can make a right triangle by drawing a line from the origin to the centre of the right-hand small circle. Then drop a line straight down to the *x*-axis.

The hypotenuse of your triangle is then just the sum of the radii of your big and little circle and the angle between the hypotenuse and the *x*-axis will be θ. Let's say that we'd like the big circle to have radius 4 and the little ones to have radius 1, in whatever units are appropriate for the software you are using. Then your triangle will look like:



Suppose you decide that you want the ears at a 45° angle, so that θ = 45°. You will have the two pieces of information about your right triangle (in addition to the right angle) that you will need to completely solve the triangle. Let's call the remaining sides *x* and *y*.



Then you'll find that

$$\sin\theta = \frac{y}{5}$$
$$5\sin\theta = y$$
$$y = 5\sin\theta = 5\sin 45^\circ \approx 3.54$$

and

$$\cos\theta = \frac{x}{5}$$
$$5\cos\theta = x$$
$$x = 5\cos\theta = 5\cos 45^\circ \approx 3.54$$

So if you chose 4 and 1 as the radii of the big and little circles, respectively, and 45° for your angle, then the coordinates of the centres of the circles will be (0,0) for the big one and (3.54, 3.54) and (−3.54, 3.54) for the two little ones.

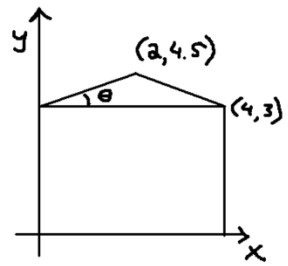
Now, for this particular setup, it's true that the *x*- and *y*-components will be the same, since we've chosen θ to be 45°. But what's nice about the analysis above is that it will still be true for any value of θ that you might wish to use, so you could experiment with a few different values and find the one that makes your diagram look like whatever you had in mind.

In fact, you could even leave your hypotenuse in terms of the two radii, $c = r_{\text{big circle}} + r_{\text{little circle}}$, and experiment with different values of the radii as well.

Let's look at another example.

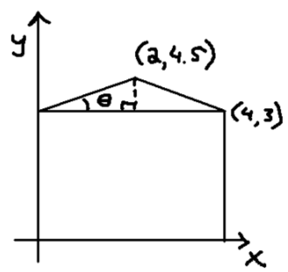
Example

Calculate the angle θ and the length of each sloping piece of roof for the house in the following diagram.

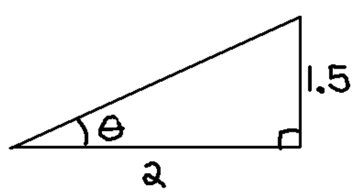


Answer:

First, let's draw a right triangle by dropping a line from the peak of the roof downwards as shown in the diagram.



By inspection, the triangle will have sides 1.5 and 2 as shown below. ("By inspection" means essentially "by looking at the diagram".)



Then we can use that

$$\tan\theta = \frac{1.5}{2}$$

$$\theta = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.87^\circ = 37^\circ$$

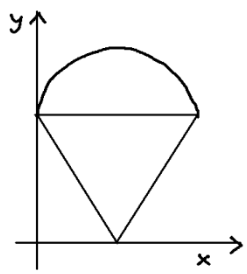
And we can calculate the remaining side using the Pythagorean theorem:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= (1.5)^2 + (2)^2 \\ c &= 2.5 \end{aligned}$$

So the angle of the roof is 37° and each piece of the sloping roof is 2.5 units long.

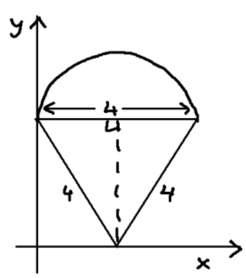
Example

A signmaker wishes to make a sign in the shape of an icecream cone, as shown in the diagram. The "cone" consists of a semicircle and an equilateral triangle. If the triangle has sides of length 4 units, calculate the coordinates of each vertex (corner) of the triangle and the radius of the semi-circle. Round, when appropriate, to two decimal places.

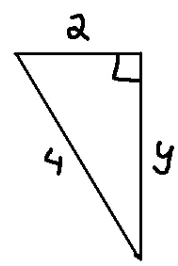


Answer:

Draw a right triangle in the diagram as shown below.



Since the figure is symmetrical, then the triangle has sides 2 and 4 as shown below.



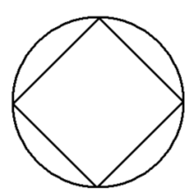
Using the Pythagorean theorem to calculate *y*:

$$a^2 + b^2 = c^2$$
$$4 + y^2 = 16$$
$$y^2 = 12$$
$$y = 2\sqrt{3} \approx 3.46$$

So the coordinates of the three vertices are (2, 0), (0, 3.46), and (4, 3.46), and the semicircle has a radius of 2 units.

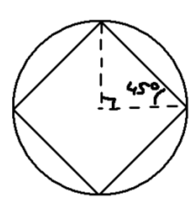
Example

A graphic is made from a square inscribed inside a circle as shown below. If the circle has a radius of 5 cm, calculate the length of the side of the square. Round your answer to two decimal places.

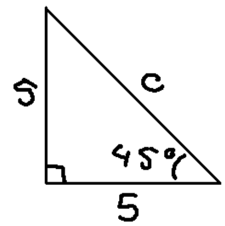


Answer

Draw a right triangle as shown below. (You could also continue the straight line down so that the diameter of the circle is the hypotenuse, but either triangle will do.)



Then two of the sides will have the same length as the radius of the circle, like so:



And then *c* can be found by Pythagorus (or trig, if you prefer):

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 25 + 25 \\ &= 50 \\ c &= \sqrt{50} = 5\sqrt{2} \approx 7.07 \end{aligned}$$

and the side of the inscribed square is 7.07 cm long.