Section 5.1: Integral Exponents and Scientific Notation

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9:33 AM

$\underbrace{\text{Integral}}_{\text{exponent that is an integer}}$

notation:

a^m ← exponent

↑ base

note: for this section, we will assume that all variables in the base are non-zero

positive exponents:

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

negative exponents:

$$x^{-5} = \frac{1}{x^5}$$

$$\frac{1}{x^{-5}} = x^5$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

$$\left(\frac{a}{b}\right)^{-3} = \left(\frac{b}{a}\right)^3 = \frac{b^3}{a^3}$$

evaluate:

$$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

$$\frac{1}{4^{-2}} = 4^2 = 16$$

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

$$-5^{-2} = \frac{-1}{5^2} = \frac{-1}{25} \quad \text{or} \quad -\frac{1}{25}$$

the product rule:

$$x^2 \cdot x^3 = (x \cdot x) \cdot (x \cdot x \cdot x) = x^5$$

in general:

$$a^m \cdot a^n = a^{m+n}$$

so $2^5 \cdot 2^7 = 2^{12}$

$2^2 \cdot 3^3 = 2^2 \cdot 3^3$

examples:

simplify:

$$3^{15} \cdot 3^{-3} = 3^{12}$$

$$-\frac{1}{2} w^{-4}\left(-6w^{-2}\right) = 3w^{-4+(-2)}$$

$$= 3w^{-6} \quad \text{or} \quad \frac{3}{w^6}$$

you can either

move all variables into numerator | give all variables with positive exponents

but don't mix the two:

rewrite $\frac{y^{-10}}{x^6}$ as either $x^{-6}y^{-10}$ or $\frac{1}{x^6 y^{10}}$

and make sure you don't have negative exponents in the denominator:

rewrite $\frac{1}{y^{-10}}$ as y^{10}

Zero exponents:

$a^0 = 1$ for a being a non-zero real number

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why? digression

$$a^n = a^n$$

$$a^n \cdot a^{-n} = a^n \cdot a^{-n}$$

$$a^{n+(-n)} = \frac{a^n}{a^n}$$

$$a^0 = 1$$

example: evaluate

$$157^0 - \left(\frac{1}{3}\right)^0 + (104)^0 - \left(\frac{3}{17}\right)^0 + 5^0 = 1$$

quotient rule:

$$\frac{a^m}{a^n} = a^{m-n}$$

why? (digression)

$$\frac{a^m}{a^n} = a^m \cdot a^{-n}$$

$$= a^{m+(-n)}$$

$$= a^{m-n}$$

examples: simplify:

$$\frac{2^7}{2^5} = 2^2 = 4$$

$$\frac{-3a^{-3}}{-21a^{-4}} = \frac{-\cancel{3}}{{}_{7}\,-\cancel{21}} \cdot \frac{a^{4}}{a^{3}} = \frac{a}{7}$$

$$= \frac{a^{-3-(-4)}}{7} = \frac{a^{1}}{7} = \frac{a}{7}$$

$$= \frac{a^{-3}a^{4}}{7} = \frac{a}{7}$$

$$\frac{2r^{-3}t^{-1}}{10r^{5}t^{2}t^{-3}} = \frac{r^{-3-5}\,t^{-1}}{5\quad t^{-1}}$$

$$= \frac{r^{-8}}{5} \quad \text{or} \quad \frac{1}{5r^{8}}$$

evaluate $(2^{-1}+2^{-1})^{-2} = \left(\frac{1}{2}+\frac{1}{2}\right)^{-2}$

$$= 1^{-2}$$
$$= 1$$

scientific notation:

evaluate

$$\frac{(6000)(0.00004)}{(30\,000)(0.002)} = \frac{6\times10^{3}\cdot 4\times10^{-5}}{3\times10^{4}\cdot 2\times10^{-3}}$$

$$= 4 \times \frac{10^{-2}}{10^{1}}$$

$$= 4 \times 10^{-3}$$