Section 7.1: cont'd

Wednesday, November 20, 2013
9:30 AM

the radical symbol

$$\sqrt[n]{a}$$

if the index n is a positive even integer and a is positive, then $\sqrt[n]{a}$ denotes the principal square root of a:

$$\sqrt{25} = 5$$

$$\sqrt[4]{16} = 2$$

if n is odd and a is real, then $\sqrt[n]{a}$ is the real n^{th} root of a:

$$\sqrt[3]{27} = 3$$

$$\sqrt[3]{-27} = -3$$

what if $a = 0$? (for any positive n)

$$\sqrt[n]{0} = 0$$

evaluate:

$\sqrt{81} = 9$

$\sqrt[4]{81} = 3$

$\sqrt[4]{-81} =$ not a real number

$\sqrt[3]{125} = 5$

$\sqrt[4]{-81}$ = not a real number

$\sqrt[3]{125}$ = 5

$\sqrt{64}$ = 8

$\sqrt[3]{64}$ = 4

$\sqrt{-64}$ = not a real number

$\sqrt[3]{-64}$ = -4

$\sqrt[6]{64}$ = 2

roots and variables:

perfect squares:

$x^2, x^4, x^6, x^8, \ldots$

how can you tell if a power of x is a perfect square? the exponent is divisible by 2

perfect cubes

$x^3, x^6, x^9, x^{12}, \ldots$

exponent divisible by 3

nitpicker-from-hell wrinkle (will not be tested)

$\sqrt{3^2} = 3$

$\sqrt{(-3)^2} = 3$

$\Rightarrow$ for all x, $\sqrt{x^2} = |x|$

from now on in this course, we'll assume all variables are non-negative and so avoid this issue

product rule:

$$\sqrt{2} \cdot \sqrt{5} = \sqrt{10}$$

$$\sqrt[3]{3} \cdot \sqrt[3]{7} = \sqrt[3]{21}$$

$$\sqrt{2} \cdot \sqrt[3]{3} = \sqrt{2}\,\sqrt[3]{3}$$

in general:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$\rightarrow$ provided that all of these roots are real numbers

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use the product rule to simplify. Assume all variables are non-negative real numbers.

$$\sqrt{12} = \sqrt{4}\,\sqrt{3}$$
$$= 2\sqrt{3}$$

$$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$$

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$$\sqrt{98} = \sqrt{2 \cdot 49} = 7\sqrt{2}$$

$$\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = 2\sqrt[3]{2}$$

$$\sqrt{28 z^6} = \sqrt{4 \cdot 7 (z^3)^2} = 2z^3\sqrt{7}$$

$$\sqrt[3]{250 y^4} = \sqrt[3]{125 \cdot 2 \; y^3 \cdot y} = 5y\sqrt[3]{2y}$$

$$\sqrt[3]{54ab^5} = \sqrt[3]{3^3 \cdot 2 \, a \, b^3 b^2}$$

$$= 3b\sqrt[3]{2ab^2}$$

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$9 \cdot 6$

$3 \cdot 3 \cdot 3 \cdot 2$

↑

if all exponents under the radical are smaller than the index, you're done

$$\sqrt[3]{48x^3y^8z^7} = \sqrt[3]{8 \cdot 6 \, x^3 \, y^6 y^2 \, z^6 z}$$

$$= 2xy^2z^2\sqrt[3]{6y^2z}$$

$$\sqrt[5]{64x^{16}y^4z^9} = \sqrt[5]{2^6 x^{16} y^4 z^9}$$

$$= \sqrt[5]{2^5 \cdot 2 \; x^{15} x \; y^4 \, z^5 z^4}$$

$$= 2x^3z\sqrt[5]{2xy^4z^4}$$