Section 7.2: Rational Exponents

Thursday, November 21, 2013
10:00 AM

fractional exponents

if n is any positive integer, then

$$a^{1/n} = \sqrt[n]{a}$$

provided that $\sqrt[n]{a}$ is real

examples: $9^{1/2} = 3$

$27^{1/3} = 3$

$(-27)^{1/3} = -3$

$(-9)^{1/2} =$ not a real number

note: $-9^{1/2} = -3$, though

$-9^{1/2} = -\sqrt{9} = -3$

$a^{m/n}$ definition:

if m and n are positive integers and $a^{1/n}$ is a real number, then

$$a^{m/n} = (a^{1/n})^m$$

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(a^m\right)^{1/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

any of these!

example:

$$9^{3/2} = \left(9^3\right)^{1/2} = 729^{1/2} = 27$$

hard to do without a calculator

$$= \left(9^{1/2}\right)^3 = 3^3 = 27$$

$$(-8)^{5/3} = \left[(-8)^{1/3}\right]^5 = (-2)^5 = -32$$

definition of $a^{-m/n}$:

$$a^{-m/n} = \frac{1}{a^{m/n}}$$

example:

$$16^{-3/4} = \frac{1}{16^{3/4}} = \frac{1}{2^3} = \frac{1}{8}$$

$$(-1000)^{-2/3} = \frac{1}{(-1000)^{2/3}} = \frac{1}{(-10)^2} = \frac{1}{100}$$

note: $$\sqrt[3]{z^{36}} = z^{36/3} = z^{12}$$