Section 7.3: Adding, Subtracting, and Multiplying Radicals

Monday, November 25, 2013
9:26 AM

examples:

$$3\sqrt{2} + 7\sqrt{2} = 10\sqrt{2}$$

why?

$$3\sqrt{2} + 7\sqrt{2}$$
$$\sqrt{2}(3 + 7)$$
$$10\sqrt{2}$$

$$3\sqrt{2} - 7\sqrt{2} = -4\sqrt{2}$$

$$3\sqrt{2} + 2\sqrt{3} = 3\sqrt{2} + 2\sqrt{3}$$

must have "like" radicals

$$3\sqrt{2} + 3\sqrt[3]{2} = 3\sqrt{2} + 3\sqrt[3]{2}$$

indices must be the same!

add:

$$\sqrt{20} + \sqrt{500} = \sqrt{4 \cdot 5} + \sqrt{100 \cdot 5}$$
$$= 2\sqrt{5} + 10\sqrt{5}$$
$$= 12\sqrt{5}$$

simplify:

$$\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$$

$$\sqrt{12} + \sqrt{24} = 2\sqrt{3} + \sqrt{4\cdot 6} = 2\sqrt{3} + 2\sqrt{6}$$

$$\sqrt[3]{2000\,w^2 z^5} - \sqrt[3]{16\,w^2 z^5}$$

$$= \sqrt[3]{2\cdot 1000\,w^2 z^3 z^2} - \sqrt[3]{2^3\cdot 2\,w^2 z^3 z^2}$$

$$= 10z\sqrt[3]{2w^2z^2} - 2z\sqrt[3]{2w^2z^2}$$

$$= 8z\sqrt[3]{2w^2z^2}$$

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multiplying:

$$\sqrt{5}\sqrt{7} = \sqrt{35}$$

$$3\sqrt{2}\left(-4\sqrt{10}\right) = -12\sqrt{20}$$

$$= -12\sqrt{4\cdot 5}$$

$$= -24\sqrt{5}$$

$$\sqrt[3]{5}\,\sqrt[3]{100} = \sqrt[3]{5\cdot 25\cdot 4}$$

$$= \sqrt[3]{5\cdot 5^2\cdot 2^2}$$

$$= \sqrt[3]{5^3 \cdot 4}$$

$$= 5 \sqrt[3]{4}$$

$$(-4\sqrt{2})^2 = (-4)^2(\sqrt{2})^2 \qquad (ab)^2 = a^2b^2$$

$$= 16 \cdot 2$$

$$= 32$$

$$\sqrt[4]{4} \cdot \sqrt[4]{8} = \sqrt[4]{2^2 \cdot 2^3}$$

$$= \sqrt[4]{2^4 \cdot 2}$$

$$= 2\sqrt[4]{2}$$

$$2\sqrt{5}(\sqrt{3} + 3\sqrt{5})$$

$$= 2\sqrt{15} + 6 \cdot 5$$

$$\left.\begin{aligned} &= 2\sqrt{15} + 30 \\ &= 30 + 2\sqrt{15} \end{aligned}\right\} \text{either}$$

convention: rational part + irrational part

first

$$(5+\sqrt{2})^2 = (5+\sqrt{2})(5+\sqrt{2})$$
$$= 25 + 10\sqrt{2} + 2$$
$$= 27 + 10\sqrt{2}$$

note:

$$(5+\sqrt{2})^2 \neq 5^2 + (\sqrt{2})^2$$

$$(\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5})$$

conjugates

$$= 6 + (\text{terms that cancel!}) - 5$$
$$= 1$$

$$(\sqrt{3}-\sqrt{2})(\sqrt{6}+1)$$
$$= \sqrt{18} + \sqrt{3} - \sqrt{12} - \sqrt{2}$$
$$= \sqrt{2\cdot 9} + \sqrt{3} - \sqrt{4\cdot 3} - \sqrt{2}$$
$$= 3\sqrt{2} + \sqrt{3} - 2\sqrt{3} - \sqrt{2}$$
$$= 2\sqrt{2} - \sqrt{3}$$

$$(\sqrt{x-1}+1)^2$$

$= x - 1 + 2\sqrt{x-1} + 1$

$= x + 2\sqrt{x-1}$

multiplying radicals with different indices:

$\sqrt{2} \cdot \sqrt[3]{2}$

$= 2^{1/2} \cdot 2^{1/3}$

$= 2^{1/2 + 1/3}$

$= 2^{3/6 + 2/6}$

$= 2^{5/6}$

$= \underbrace{\sqrt[6]{2^5} \quad \text{or} \quad \sqrt[6]{32}}_{\text{either}}$

write each product as a single radical:

$\sqrt{3} \cdot \sqrt[4]{3} = 3^{1/2} \cdot 3^{1/4}$

$= 3^{3/4}$

$= \sqrt[4]{3^3} \quad \text{or} \quad \sqrt[4]{27}$

$\sqrt{2} \cdot \sqrt[3]{3} = 2^{1/2} \cdot 3^{1/3}$

$$= 2^{3/6} \cdot 3^{2/6}$$

$$= \left(2^3 \cdot 3^2 \right)^{1/6}$$

$$= \sqrt[6]{2^3 \cdot 3^2} = \sqrt[6]{72}$$

$$\sqrt[3]{2m} \cdot \sqrt[4]{2n}$$

$$= (2m)^{1/3} \cdot (2n)^{1/4}$$

$$= (2m)^{4/12} \cdot (2n)^{3/12}$$

$$= 2^{4/12} \, m^{4/12} \, 2^{3/12} \, n^{3/12}$$

$$= \left(2^4 \, m^4 \, 2^3 \, n^3 \right)^{1/12}$$

$$= \sqrt[12]{2^7 m^4 n^3}$$