

Section 7.5: Solving Equations with Radicals and Exponents

Tuesday, November 26, 2013
9:28 AM

Assignment #6 due on
Tuesday, Dec 3

the odd-root property:

if n is an odd positive integer, then

$$x^n = k$$

is equivalent to

$$x = \sqrt[n]{k}$$

for any real number k

so if $x^3 = 8$, then $x = \sqrt[3]{8} = 2$

$x^3 = -8$ $x = \sqrt[3]{-8} = -2$

examples: solve

$$x^5 = -32$$

$$x = -2$$

$$\{-2\}$$

$$(y-3)^3 = 64$$

$$y-3 = 4$$

$$y = 7$$

$$\{7\}$$

$$z^3 = 40$$

$$z = \sqrt[3]{40}$$

$$= \sqrt[3]{8 \cdot 5}$$

$$= 2 \sqrt[3]{5}$$

$$\{2 \sqrt[3]{5}\}$$

the even-root property:

$x^2 = 9$ has two solutions

$$x = \pm 3$$

suppose n is a positive even integer

\rightarrow if $k > 0$, then $x^n = k$ is equivalent to

$$x = \pm \sqrt[n]{k}$$

→ if $k=0$, then $x^n=0$ means $x=0$

→ if $k<0$, then $x^n=k$ has no real solutions

examples:

$$x^2 = 4$$

$$x = \pm \sqrt{4}$$

$$= \pm 2$$

$$\{\pm 2\}$$

$$y^2 - 40 = 0$$

$$y^2 = 40$$

$$y = \pm \sqrt{40}$$

$$= \pm 2\sqrt{10}$$

$$\{\pm 2\sqrt{10}\}$$

$$w^2 + 49 = 0$$

$$w^2 = -49$$

no real solutions

$$(w+3)^2 - 12 = 0$$

$$(w+3)^2 = 12$$

$$w+3 = \pm 2\sqrt{3}$$

$$w = -3 \pm 2\sqrt{3}$$

$$\{-3 \pm 2\sqrt{3}\}$$

equations involving radicals:

solve

$$\sqrt{b-1} - 5 = 1$$

isolate \rightarrow
the
radical

$$(\sqrt{b-1})^2 = (6)^2$$

$$b-1 = 36$$

$$b = 37$$

$$\{37\}$$

check: $\sqrt{36} - 5 = 1 \quad \checkmark$

$$\sqrt{x^2 - x - 4} = 4$$

$$x^2 - x - 4 = 16$$

$$x^2 - x - 20 = 0$$

$$(x-5)(x+4) = 0$$

$$x = 5, -4$$

$$\{-4, 5\}$$

check: $x = 5$

$$\sqrt{25 - 5 - 4} = 4$$
$$\sqrt{16} = 4 \checkmark$$

$x = -4$

$$\sqrt{16 + 4 - 4} = 4 \checkmark$$

$$\sqrt{2x^2 + 6x + 9} = x$$

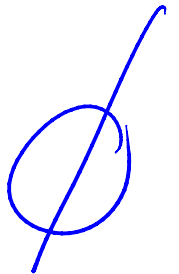
$$2x^2 + 6x + 9 = x^2$$

$$x^2 + 6x + 9 = 0$$

$$(x+3)(x+3) = 0$$

$$x = \cancel{-3}$$

extraneous



check:

$$\sqrt{18 - 18 + 9} = -3$$
$$3 = -3$$

$\sqrt{\quad}$ means the \rightarrow

X

principal square root
(the positive one)

MORAL OF THE STORY:

CHECK YOUR WORK!

Solve:

$$\sqrt{x} + \sqrt{x+3} = 3$$

$$(\sqrt{x+3})^2 = (3 - \sqrt{x})^2$$

$$\cancel{x} + 3 = 9 - 6\sqrt{x} + \cancel{x}$$

$$6\sqrt{x} = 6$$

$$\sqrt{x} = 1$$

$$x = 1$$

{1}

$$\text{check: } \sqrt{1} + \sqrt{4} = 3 \quad \checkmark$$

$$\sqrt{2x+1} - \sqrt{x} = 1$$

$$(\sqrt{2x+1})^2 = (1 + \sqrt{x})^2$$

✓ ✓

$$2x+1 = 1 + 2\sqrt{x} + x$$

$$(x)^2 = (2\sqrt{x})^2$$

$$x^2 = 4x$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0, 4$$

$$\{0, 4\}$$

check: $x=0$

$x=4$

$$\sqrt{1} - \sqrt{0} = 1 \quad \checkmark$$

$$\sqrt{9} - \sqrt{4} = 1 \quad \checkmark$$