Section 7.6: cont'd

Thursday, November 28, 2013
9:30 AM

recall: complex number $a + bi$

a and b are real

$i = \sqrt{-1}$ and $i^2 = -1$

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addition and subtraction:

$$(-3+7i) + (5-4i) = 2 + 3i$$

$$(-3+7i) - (5-4i) = -8 + 11i$$

multiplication: (recall: $i^2 = -1$)

$$(-3+7i)(5-4i)$$
$$= -15 + 12i + 35i - 28i^2$$
$$= -15 + 47i - 28(-1)$$
$$= -15 + 47i + 28$$
$$= 13 + 47i$$

Convention: write real part first

$$(-2+3i)(5-i)$$
$$= -10 + 2i + 15i - 3i^2$$
$$= -10 + 17i + 3$$
$$= -7 + 17i$$

$$(3+2i)(3-2i)$$

$(3+2i)(3-2i)$

Complex conjugates

$= 9 + (\text{terms that cancel}) - 4i^2$

$= 9 + 4$

$= 13$

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powers of complex numbers:

$i = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

$i^5 = i$
$i^6 = -1$
$i^7 = -i$
$i^8 = 1$

$i^9 = i$
$i^{10} = -1$
$i^{11} = -i$
$i^{12} = 1$

$i^3 = i^2 \cdot i$
$= -1 \cdot i$
$= -i$

$i^4 = i^2 \cdot i^2$
$= (-1)(-1)$
$= 1$

note: $i^{100} = 1$ because 100 is a multiple of 4

so $i^{100} = (i^4)^{25}$

simplify:

$(-3i)^3 = (-3)^3 i^3 = -27(-i) = 27i$

$(2i)^4 = 2^4 i^4 = 16(1) = 16$

$i^{27} = i^{24} i^3 = (1)(-i) = -i$

$i^{52} = (i^4)^{13} = 1^{13} = 1$

$i^{74} = i^{72} i^2 = (1)(-1) = -1$

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division:

$$\frac{1+i}{2-3i}\left(\frac{2+3i}{2+3i}\right) = \frac{2+3i+2i+3i^2}{4-9i^2}$$

$$= \frac{2+5i-3}{4+9}$$

$$= \frac{-1+5i}{13} \quad \text{or} \quad \frac{-1}{13} + \frac{5i}{13}$$

note: $\frac{5+3i}{5} = 1 + \frac{3}{5}i$

$\frac{5+10i}{5} = 1+2i$

$$\frac{5-i}{5+i}\left(\frac{5-i}{5-i}\right) = \frac{25-10i+i^2}{25-i^2}$$

$$= \frac{25-10i-1}{25+1}$$

$$= \frac{24-10i}{}$$

$$= \frac{24 - 10i}{26}$$

$$= \frac{12 - 5i}{13}$$

Square roots of negative numbers:

$$\sqrt{-4} = \sqrt{4}\sqrt{-1}$$
$$= 2i$$

$$\sqrt{-25} = \sqrt{25}\sqrt{-1}$$
$$= 5i$$

$$\sqrt{-7} = \sqrt{7}\sqrt{-1}$$
$$= i\sqrt{7}$$

$$\sqrt{-8} = \sqrt{8}\sqrt{-1}$$
$$= 2\sqrt{2}\, i$$
$$= 2i\sqrt{2}$$

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solving equations:

solve $x^2 = -9$

$x = \pm\sqrt{-9}$ ← square root

$x = \pm\sqrt{-9}$ ← square root both sides

$= \pm\sqrt{9}\sqrt{-1}$

$= \pm 3i$

Solve $y^2 + 4 = 0$

$y^2 = -4$

$y = \pm\sqrt{-4}$

$= \pm 2i$ $\{\pm 2i\}$

Solve

$5x^2 + 3 = 0$

$5x^2 = -3$

$x^2 = \frac{-3}{5}$

$x = \pm\sqrt{\frac{-3}{5}}$

$= \pm i\sqrt{\frac{3}{5}}\sqrt{\frac{5}{5}}$

$= \pm i\sqrt{15}$ $\{\pm i\sqrt{15}\}$

$= \pm \frac{i\sqrt{15}}{5}$ $\left\{ \pm \frac{i\sqrt{15}}{5} \right\}$