Section 8.1: Cont'd

Tuesday, December 03, 2013
8:28 AM

Completing the Square:

$x^2 + 12x + \underline{36} = (x+6)^2$

$x^2 - 8x + \underline{16} = (x-4)^2$

$x^2 + 5x + \underline{\frac{25}{4}} = (x+5/2)^2$

Solve by factoring and by completing the square:

factoring:

$x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$x = 7, -1$

completing the square:

$x^2 - 6x - 7 = 0$

$x^2 - 6x + \underline{9} = 7 + \underline{9}$

Square root both sides →

$(x-3)^2 = 16$

$x - 3 = \pm 4$

$x = 3 \pm 4$

$= -1, 7$

Solve:

$$x^2 + 10x - 3 = 0$$

$$x^2 + 10x + \underline{25} = 3 + \underline{25}$$

Square root → both sides

$$(x+5)^2 = 28$$

$$x + 5 = \pm\sqrt{28}$$

$$x + 5 = \pm 2\sqrt{7}$$

$$x = -5 \pm 2\sqrt{7}$$

check: $x = -5 + 2\sqrt{7}$

$$x^2 + 10x - 3 = 0$$

$$(-5+2\sqrt{7})^2 + 10(-5+2\sqrt{7}) - 3 = 0$$

$$25 - \cancel{20\sqrt{7}} + 4\cdot 7 - 50 + \cancel{20\sqrt{7}} - 3 = 0$$

$$25 + 28 - 50 - 3 = 0$$

$$0 = 0 \quad \checkmark$$

Solve:

$$x^2 - 7x + 4 = 0$$

$$x^2 - 7x + \underline{\tfrac{49}{4}} = -4 + \underline{\tfrac{49}{4}}$$

$$(x - 7/2)^2 = \tfrac{-16}{4} + \tfrac{49}{4}$$

Square → $(x - 7/2)^2 = \frac{33}{4}$

square root both sides →

$$(x - 7/2)^2 = \frac{33}{4}$$

$$x - 7/2 = \pm\sqrt{\frac{33}{4}}$$

$$x = \frac{7}{2} \pm \frac{\sqrt{33}}{2}$$

or $\frac{7 \pm \sqrt{33}}{2}$

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what if the leading coefficient isn't 1? make it 1.

$$3w^2 + 4w - 1 = 0$$

$$w^2 + \frac{4}{3}w - \frac{1}{3} = 0$$

$$w^2 + \frac{4}{3}w + \left(\frac{2}{3}\right)^2 = \frac{1}{3} + \left(\frac{2}{3}\right)^2$$

$\frac{4}{3} \div 2 = \frac{4}{3} \cdot \frac{1}{2} = \boxed{\frac{2}{3}}$

$$w^2 + \frac{4}{3}w + \frac{4}{9} = \frac{1}{3} + \frac{4}{9}$$

$$\left(w + \frac{2}{3}\right)^2 = \frac{3}{9} + \frac{4}{9}$$

square root → $\left(w + \frac{2}{3}\right)^2 = \frac{7}{9}$

$$w + \frac{2}{3} = \pm\sqrt{\frac{7}{9}}$$

$$w + \frac{2}{3} = \pm \frac{\sqrt{7}}{3}$$

$$w = \frac{-2 \pm \sqrt{7}}{3}$$

$$x^2 + 4x + 5 = 0$$

$$x^2 + 4x + \underline{4} = -5 + \underline{4}$$

$$(x+2)^2 = -1$$

$$x + 2 = \pm \sqrt{-1}$$

$$x + 2 = \pm i$$

$$x = -2 \pm i$$

check: $x = -2 + i$

$$x^2 + 4x + 5 = 0$$

$$(-2+i)^2 + 4(-2+i) + 5 = 0$$

$$4 - \cancel{4i} + i^2 - 8 + \cancel{4i} + 5 = 0$$

$$4 - 1 - 8 + 5 = 0$$

$$0 = 0 \quad \checkmark$$

$$2y^2 - 3y + 2 = 0$$

$$y^2 - \frac{3}{2}y + 1 = 0$$

$$y^2 - \frac{3}{2}y + \left(\frac{3}{4}\right)^2 = -1 + \left(\frac{3}{4}\right)^2$$

$$y^2 - \frac{3}{2}y + \frac{9}{16} = -1 + \frac{9}{16}$$

$$\left(y - \frac{3}{4}\right)^2 = \frac{-7}{16}$$

$$y - \frac{3}{4} = \pm\sqrt{\frac{-7}{16}}$$

$$y = \frac{3}{4} \pm \frac{i\sqrt{7}}{4}$$

$$= \frac{3 \pm i\sqrt{7}}{4}$$

} either