Section 8.2: The Quadratic Formula

Tuesday, December 03, 2013
9:30 AM

complete the square:

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a}\left(\frac{4a}{4a}\right) + \frac{b^2}{4a^2}$$

$$= \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

square root →

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Use the quadratic formula to solve:

$$2x^2 - 3x + 2 = 0$$

$a = 2 \qquad b = -3 \qquad c = 2$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{3 \pm \sqrt{9 - 4 \cdot 2 \cdot 2}}{2 \cdot 2}$$

$$= \frac{3 \pm \sqrt{9-16}}{4}$$

$$= \frac{3 \pm \sqrt{-7}}{4}$$

$$= \frac{3 \pm i\sqrt{7}}{4}$$

Solve: $\quad m^2 + 2m = 8$

$$m^2 + 2m - 8 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4 \cdot 1(-8)}}{2}$$

$$= \frac{-2 \pm \sqrt{4 + 32}}{2}$$

$$= \frac{-2 \pm \sqrt{36}}{2}$$

$$= \frac{-2 \pm 6}{2}$$

$$= -1 \pm 3$$

$$= -4, 2$$

solve $\quad 4y^2 - 12y + 9 = 0$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{12 \pm \sqrt{144 - 4 \cdot 4 \cdot 9}}{8}$$

$$= \frac{12 \pm \sqrt{144 - 144}}{8}$$

$$= \frac{12 \pm 0}{}$$

$$= \frac{12 \pm 0}{8}$$

$$= \frac{12}{8} = \frac{3}{2}$$

note $4y^2 - 12y + 9 = 0$

$$(2y-3)^2 = 0$$

$$y = \frac{3}{2}$$

$$p^2 + 6p + 4 = 0$$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{36 - 16}}{2}$$

$$= \frac{-6 \pm \sqrt{20}}{2}$$

$$= \frac{-6 \pm 2\sqrt{5}}{2}$$

$$= -3 \pm \sqrt{5}$$

number of real solutions:

when do you get real solutions?

→ when $\underbrace{b^2-4ac}_{\text{discriminant}}$ is ≥ 0

note: when $b^2-4ac > 0$, get two real solutions

when $b^2-4ac = 0$, get one real solution

example: Calculate the discriminant and state the number of real solutions for

a) $v^2 = 3v + 5$

$v^2 - 3v - 5 = 0$

$$b^2-4ac = 9 - 4(-5)$$
$$= 9 + 20$$
$$= 29$$

↑ greater than zero

∴ two real solutions

b) $25x^2 - 20x + 4 = 0$

$$b^2-4ac = 400 - 4(25)(4)$$

$= 400 - 400$

$= 0$

$\therefore$ one real solution

c) $-y^2 + 3y - 4 = 0$

$b^2 - 4ac = 9 - (4)(-1)(-4)$

$= 9 - 16$

$= -7$

$\therefore$ no real solutions

Find two positive real numbers that differ by 2 and have a product of 10.

let $x =$ first number

$x + 2 =$ second number

$x(x+2) = 10$

$x^2 + 2x = 10$

$x^2 + 2x - 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4(-10)}}{2}$$

$$= \frac{-2 \pm \sqrt{44}}{2}$$

$$= \frac{-2 \pm 2\sqrt{11}}{2}$$

$$= -1 \pm \sqrt{11}$$

first number $\quad x = \underbrace{\cancel{-1-\sqrt{11}}}_{\text{negative}}, \ -1+\sqrt{11}$

second number $\quad x + 2 = -1 + \sqrt{11} + 2$

$$= 1 + \sqrt{11}$$

The two numbers are $-1 + \sqrt{11}$ and $1 + \sqrt{11}$.

$$(-1 + \sqrt{11})(1 + \sqrt{11}) = -1 + 11 = 10$$