Section 8.3: More About Quadratic Equations

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9:34 AM

recall from yesterday:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

writing a quadratic with given solutions:

example: write a quadratic whose solution set is $\{\pm\sqrt{3}\}$. (Answers may vary.)

$$(x - \sqrt{3})(x + \sqrt{3}) = 0$$

$$x^2 - 3 = 0$$

$$4x^2 - 12 = 0$$

$$4x^2 + 2x - 12 = 2x$$

any of these

note: $x = \pm\sqrt{3}$ is not quadratic

write an equation whose solution set is $\{\pm 5i\}$

$$x^2 + 25 = 0$$

$$(x - 5i)(x + 5i) = 0$$

Using the discriminant in factoring:

how can you tell if a polynomial can be factored?

→ use the discriminant! $b^2 - 4ac$

example: can $2x^2 + 3x - 5$ be factored?

cannot solve this quadratic because it's not an equation

but let's calculate the discriminant:

$$b^2 - 4ac = 9 - 4(2)(-5)$$
$$= 9 + 40$$
$$= 49$$

→ if the discriminant is a perfect square, then the quadratic can be factored example: can $3x^2 + 5x - 1$ be factored?

$$b^2 - 4ac = (5)^2 - 4(-1)(3)$$
$$= 25 + 12$$
$$= 37$$

not a perfect square

$\therefore$ no

equations quadratic in form:

solve: $2x - 5\sqrt{x} + 2 = 0$

let $m = \sqrt{x}$

$m^2 = x$

$$2m^2 - 5m + 2 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 - 4 \cdot 2 \cdot 2}}{4}$$

$$= \frac{5 \pm \sqrt{25 - 16}}{4}$$

$$\frac{\qquad}{4}$$

$$= \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4}$$

$$m = \frac{1}{2},\ 2$$

$$\sqrt{x} = \frac{1}{2},\ 2$$

$$x = \frac{1}{4},\ 4$$

check: $x = 4$:

$$2x - 5\sqrt{x} + 2 = 0$$
$$2\cdot 4 - 5\sqrt{4} + 2 = 0$$
$$8 - 10 + 2 = 0$$
$$0 = 0 \checkmark$$

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solve $x^{2/3} - x^{1/3} - 20 = 0$

method #1:

let $y = x^{1/3}$

$y^2 = x^{2/3}$

$$y^2 - y - 20 = 0$$

$$(y-5)(y+4) = 0$$

$$x^{1/3} = y = 5,\ -4$$

$x = 125, -64$

$\{-64, 125\}$

note: $x^{1/3} = 5$

$\sqrt[3]{x} = 5$ ← cube both sides

$x = 5^3$

method #2:

$x^{2/3} - x^{1/3} - 20 = 0$ $ac = -20$, $+4\ -5$

$x^{2/3} + 4x^{1/3} - 5x^{1/3} - 20 = 0$

$x^{1/3}(x^{1/3} + 4) - 5(x^{1/3} + 4) = 0$

$(x^{1/3} - 5)(x^{1/3} + 4) = 0$

$x^{1/3} = -4, 5$

$x = -64, 125$

method #3:

$$x^{1/3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

etc