Math 173 – Assignment #2 **Name:** Solution Set

Total: 50

1. For $f(x)=\frac{1}{x}+3$ and $g(x)=\frac{1}{x-3}$, find $(f\circ g)(x)$ and $(g\circ f)(x)$.

$$(f\circ g)(x) = f(g(x)) = f\left(\frac{1}{x-3}\right) = \frac{1}{1/x-3}+3 = x-3+3 = x$$

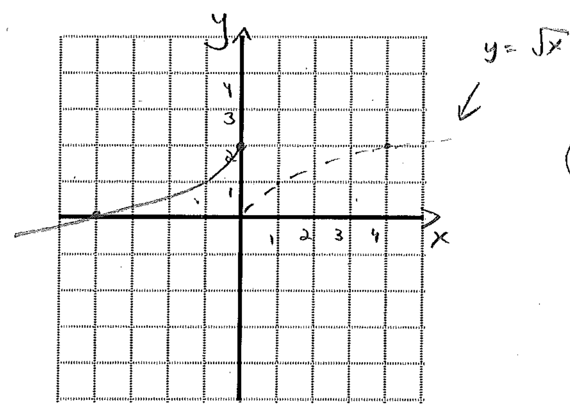
$$(g\circ f)(x) = g(f(x)) = g\left(1/x+3\right) = \frac{1}{\frac{1}{x}+3-3} = \frac{1}{1/x} = x$$

3

2. Sketch the graph of the following function. Clearly label at least two points on your graph. (5 points)

$f(x)=2-\sqrt{-x}$

this is $f(x)=\sqrt{x}$ reflected in both x & y and then shifted up by 2



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3. Write an equation for a function that has a graph with the shape of $y=|x|$ but upside-down and shifted left 1 unit and down 3 units.

upside down: $y=-|x|$

and shifted: $\boxed{y=-|x+1|-3}$

2

4. Is the function $f(x)=\dfrac{x}{x^2+1}$ even, odd, or neither even nor odd? Show your work.

$$f(-x)=\frac{-x}{(-x)^2+1}$$
$$=\frac{-x}{x^2+1}$$
$$=-f(x)$$

$\therefore$ odd

2

5. Calculate the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for the function $f(x)=5x^2$.

$$dq=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{5(x+h)^2-5x^2}{h}$$
$$=\frac{\cancel{5x^2}+10xh+5h^2-\cancel{5x^2}}{h}$$
$$=\frac{10xh+5h^2}{h}=10x+5h \quad \text{or} \quad 5(2x+h)$$

3

6. Find the vertex for the following parabola. Is the vertex at the maximum or minimum point in the parabola?

$f(x)=\dfrac{1}{4}x^2+3x-3$ $\leftarrow$ leading coeff is +, so $\cup$ and vertex at min

$(-6,-12)$

3

minimum

method #1:

nifty trick:

$$x=-\frac{b}{2a}=\frac{-3}{1/2}=-6$$

$$f(-6)=\tfrac14(-6)^2+3(-6)-3$$
$$=-12$$

method #2:

completing the square:

$$f(x)=\tfrac14(x^2+12x+\_\_)-3-\_\_$$
$$=\tfrac14(x^2+12x+36)-3-9$$
$$=\tfrac14(x+6)^2-12$$

so vertex is $(-6,-12)$

7. Rewrite the equation of the following parabola in the form $f(x)=a(x-h)^2+k$ by completing the square. Then state the axis of symmetry and the coordinates of the vertex, and sketch the graph (as accurately as possible!).

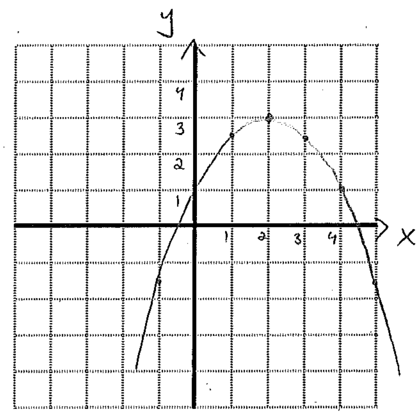
$$f(x)=-\frac{1}{2}x^2+2x+1$$

$= -\tfrac{1}{2}(x^2-4x+\_\_)+1-\_\_$

$= -\tfrac{1}{2}(x^2-4x+4)+1-(-\tfrac{1}{2})(4)$

$= -\tfrac{1}{2}(x-2)^2+1+2$

$= -\tfrac{1}{2}(x-2)^2+3$



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equation: $f(x)=-\tfrac{1}{2}(x-2)^2+3$

vertex: $(2, 3)$

axis of symmetry: $x=2$

8. Factor the following polynomial into linear factors.

$$f(x)=x^3-4x-15$$

$f(x)=(x-3)\left(x+\frac{3+i\sqrt{11}}{2}\right)\left(x+\frac{3-i\sqrt{11}}{2}\right)$ 1

rational zeros theorem:

$\frac{p}{q}=\frac{\pm1, \pm3, \pm5, \pm15}{\pm1}=\pm1, \pm3, \pm5, \pm15$

optional: Descartes: $f(-x)=-x^3+4x-15$
positive real zeros = 1
negative real zeros = 2, 0

so will try positive answers first.

$f(1)=-18$
$f(3)=0$

so $x-3$ is a factor 1

$$\begin{array}{r} x^2+3x+5 \\ x-3\overline{)x^3+0x^2-4x-15} \\ \underline{x^3-3x^2} \\ 3x^2-4x \\ \underline{3x^2-9x} \\ 5x-15 \end{array}$$ 1

x^2+3x+5 cannot be factored (into rational numbers),

4

so use quadratic formula:

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-3\pm\sqrt{9-20}}{2}$ 1

$=\frac{-3\pm\sqrt{-11}}{2}=\frac{-3\pm i\sqrt{11}}{2}$

so

$f(x)=(x-3)\left(x-\left(\frac{-3+i\sqrt{11}}{2}\right)\right)\left(x-\left(\frac{-3-i\sqrt{11}}{2}\right)\right)$

$=(x-3)\left(x+\frac{3}{2}-\frac{i\sqrt{11}}{2}\right)\left(x+\frac{3}{2}+\frac{i\sqrt{11}}{2}\right)$

9. Consider the following polynomial: $f(x) = -(x+1)^2(x-1)^2(x-3)$

a) Find the zeros of this polynomial and their multiplicities.

$x = -1, +1, +3$ ← mult 1

both mult 2

[note: touches at $(-1,0)$ and $(1,0)$, crosses at $(3,0)$] (2)

b) Find the y-intercept.

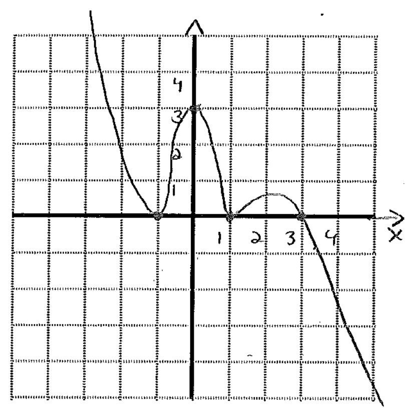
$f(0) = -(1)^2(-1)^2(-3) = 3$ so $(0, 3)$ (1)

c) Sketch the graph.

end behavior:

leading term test:

↑ coeff is −, degree is odd ↓



(3)

10. Use the Rational Zeros Theorem to list all possible rational zeros of *f(x)*.

$f(x) = 4x^5 - 3x^3 + 5x^2 - 6$

(2)

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4} : \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

11. Using Descartes' Rule, how many positive real zeros and negative real zeros can the following polynomial have? Do not solve it!

$f(x) = x^5 + 5x^4 + x^3 - 3x^2 + 4x + 7$

positive real zeros: 2, 0

2 changes

negative real zeros: 3, 1

(2)

$f(-x) = -x^5 + 5x^4 - x^3 - 3x^2 - 4x + 7$

3 changes

12. Consider the following rational function:

$$f(x) = \frac{x^2}{x^2 - 4}.$$

a) What is the y-intercept? (set $x = 0$)

$f(0) = \frac{0}{-4} = 0$ $\boxed{(0,0)}$ (1)

b) What are the x-intercepts? (set num $= 0$)

$x^2 = 0$
$x = 0$

$\boxed{(0,0)}$ mult of 2 (touch, not cross) (1)

c) Are there any vertical asymptotes? If so, where? (set denom $= 0$)

$x^2 - 4 = 0$
$x = \pm 2$

$\boxed{x = \pm 2}$ (1)

d) Are there any horizontal asymptotes? If so, where?

yes, degree of num = degree of denom
y = ratio of leading coeffs

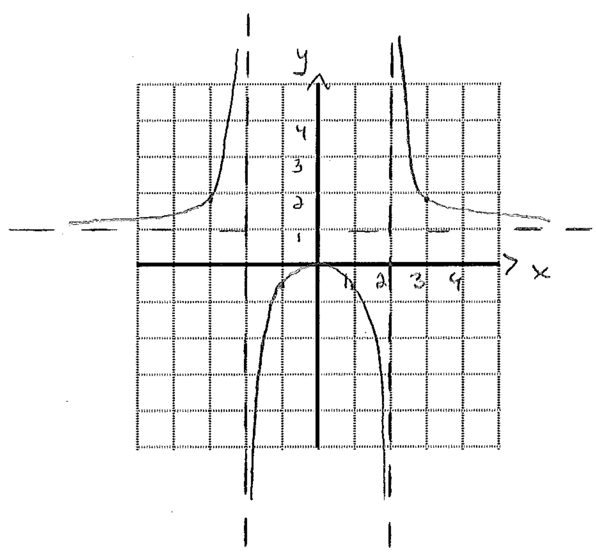
$\boxed{y = 1}$ (1)

e) Are there any slant asymptotes? If so, where?

no (can't have both horiz & oblique) (1).

f) Sketch the graph as accurately as possible.

x	y
1	$-1/3 = -0.\bar{3}$
3	$9/5 = 1.8$



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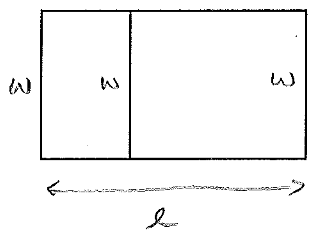
13. A fourth-grade class decides to enclose a rectangular garden and then divide the garden into two sections by an additional piece of fence, as shown in the diagram. What is the maximum area that the class can enclose with 24 feet of fence?



$P = 2l + 3w$

$24 = 2l + 3w$

$24 - 3w = 2l$

$l = \frac{24-3w}{2}$

5

$A = lw$

$A = \left(\frac{24-3w}{2}\right)w = 12w - \frac{3}{2}w^2$

method #1:

this is a parabola with vertex at

$w_{max} = \frac{-b}{2a} = \frac{-12}{-3} = 4$

$l_{max} = \frac{24-12}{2} = 6$

$A_{max} = l_{max} w_{max}$

$= 24$

method #2:

completing the square:

$A = -\frac{3}{2}w^2 + 12w$

$= -\frac{3}{2}(w^2 - 8w + \_\_) - \_\_\_\_$

$= -\frac{3}{2}(w^2 - 8w + 16) - (-\frac{3}{2})(16)$

$= -\frac{3}{2}(w-4)^2 + 24$

parabola with vertex at

$(\underset{w_{max}}{4}, \underset{A_{max}}{24})$

method #3:

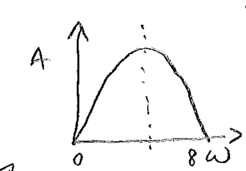
$A = w(12 - \frac{3}{2}w)$

parabola with zeros at

$w = 0$ and $12 - \frac{3}{2}w = 0$

$12 = \frac{3}{2}w$

$w = 8$



leading coeff is −

so w_{max} is halfway between 0 and 8,

so $w_{max} = 4$

$\boxed{\text{The maximum area that can be enclosed is 24 square feet.}}$